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STARK EFFECT FOR THE SPECTRA OF SILVER, COPPER, AND GOLD

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ABSTRACT

Stark effect for the spectral lines of silver, copper, and gold were examined by Lo Surdo's method. The results are given separately for each metal.

Silver.—Stark effect of the *doublet diffuse series* lines were observed up to 2p—7d. Their behavior, together with the results already obtained by Takamine, are discussed in detail, and *the components are classified as forbidden combinations. Similarity* between *silver* and *orthohelium* lines *is noticed.*

Copper.—Stark effect for the lines of the diffuse series were observed up to 2p—6d. They behave more simply in the electric field than silver lines and resemble rather those of zinc and cadmium, but *none of the 2p–mp combinations have been observed.*

Gold.—The effect was observed *only for the 2p—4d lines.* The behavior of this group is quite similar to that of the 2p—4d group of silver.

I. INTRODUCTION

In 1918 Takamine[1] examined the Stark effect for the spectrum lines of various metals using Lo Surdo's method. In that work, however, new lines were not assigned to series owing to the lack of data on term values. In the present paper an attempt is made to interpret Takamine's results, and also certain extensions are made for higher members of the series.

The experimental arrangements were the same as described in a recent work of Y. Fujioka[2] dealing with the Stark effect for zinc and cadmium. As mentioned in that paper, particular care was taken to use as the window of the discharge tube a crystalline quartz plate

[1] *Astrophysical Journal*, **50**, 23, 1919.

[2] *Scientific Papers of the Institute of Physical and Chemical Research*, **5**, 45, 1926.

cut parallel to the optical axis, and mounted so as to make the directions of the axis coincide with the electric field.

The cathode was made of the metal to be examined. The residual gas of the discharge tube was in most cases helium (pressure, 3–5 mm). Sometimes when chlorine was used, the cathode dark space was relatively shorter, so that it was necessary to reduce the pressure (1 mm) to obtain a sufficient length of the dark space.

Hilger's two prism U.-V. spectrograph was generally used for obtaining intense photographs of the region from the visible down to 3300 A, the region covering most of the diffuse series lines of silver, copper, and gold. Use was also made of Hilger's quartz spectrograph (model E_2).

Since most of the new lines appear in the close neighborhood of the diffuse series lines, the measurements of the former were made relatively to the latter. The classification of the different new lines coming out in an electric field as forbidden combinations was originally carried out successfully in the beautiful work of H. M. Hansen, T. Takamine, and S. Werner[1] for mercury. As will be seen later, the features observed on silver, copper, and gold in the present investigation are in many respects analogous to those observed for mercury.

II. STARK EFFECT FOR SILVER LINES

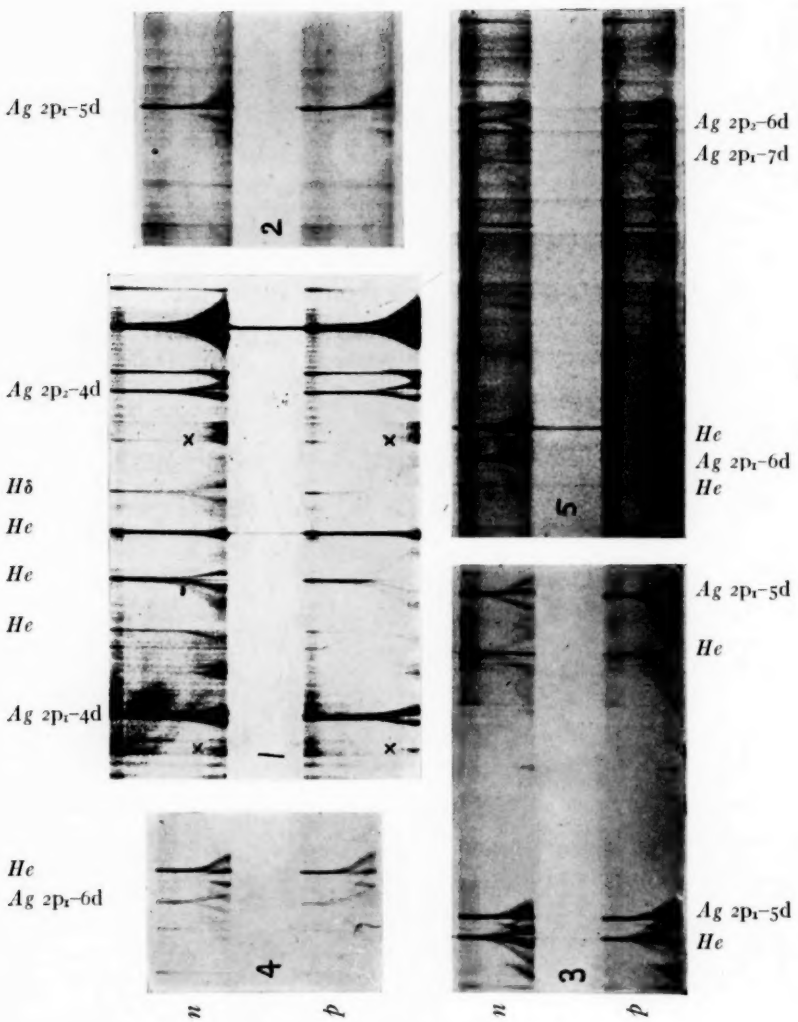
Each line of the diffuse series of silver seems to resolve into several components in an electric field, some of them being detached from the main line. They will now be described in separate groups.

$2p_1-4d$ (λ 4211) group.—With regard to this group, Takamine's experiment showed more detail than the present one. His result is diagrammatically shown in Figure 1. Besides the lines given in this figure, unaffected lines of $2p_1-4d_1$ and $2p_1-4d_2$ are observed in the electric field in both *n*- and *p*-patterns. These lines, as well as the unaffected lines $2p_2-4d_2$ and $2p_1-5d_1$, are probably due to the emission from the part of the source unaffected by the field. In the present experiment these unaffected components were not observed with appreciable intensity.

$2p_1-4d_2$ (λ 4212.60).—Faint and particularly so in the electric field; displaced toward red.

[1] *Det. Kgl. Danske Videnskabernes Selskab. Matem.-fis.*, **3**, 1923.

PLATE IV



STARK EFFECT FOR THE SPECTRUM OF SILVER

1 – E_{max} = 4.3 × 10⁴ V/cm; 2 – E_{max} = 4.0 × 10⁴ V/cm in chlorine; 3, 4, 5 – E_{max} = 4.5 × 10⁴ V/cm.

$2p_1-4d_1$ (λ *4210.71*).—Rather enhanced in the field; displaced toward violet. Takamine observed the *n*-component to be resolved into two branches but the one on the violet side seems doubtful (perhaps due to the contamination of the strong *p*-component).

A new line, presumably $2p_1-4f$, appears slightly on the violet side of $2p_1-4d_2$ and is displaced toward red nearly parallel to this

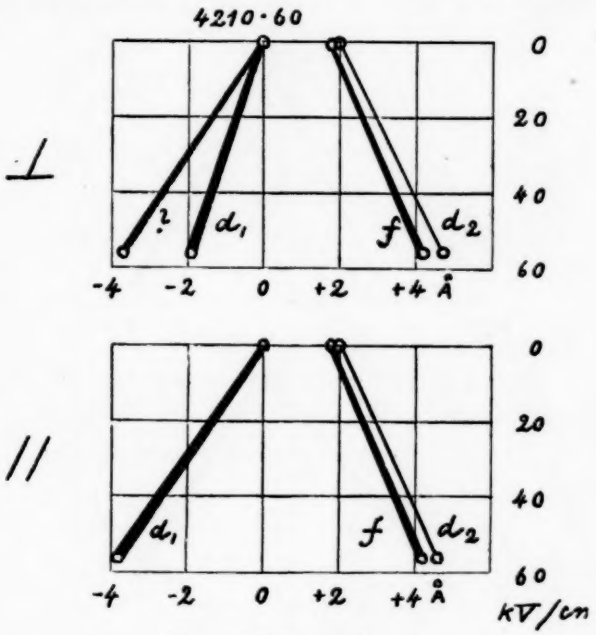


FIG. 1.—Silver $2p_1$–4d group

line. 4f given in A. Fowler's *Report*[1] exceeds $4d_2$(6891.4) by 0.8 cm^{-1}, while from the foregoing new line (in the weakest field) the difference is 1.0 cm^{-1}.

Another new line, which is not given in Figure 1, is observed on the red side of $2p_1-4d_2$, and is displaced toward the red. Its wavelength was measured to be 4226.55 A by Takamine and 4226.9 in the present experiment, while the calculated value of $2p_1-4p_1$ is 4224.43.

$2p_2-4d$ (λ *4055*) *group*.—In this group the main line is displaced toward the red. A component, nearly as strong as the main line,

[1] *Report on Series in Line Spectra*, p. 112, 1923.

starts on the side of the shorter wave-length, 0.2 A from the main line, and is displaced toward the violet (Fig. 2). Probably the main line is $2p_2-4d_2$ and the detached one $2p_2-4f$. Here the difference $4d_2-4f$ is found to be 1.2 cm^{-1}, in very good agreement with the case of $2p_1-4f$ combination. One peculiar feature may be noted in that the line $2p_1-4f$ is displaced toward the red, whereas the line $2p_2-4f$ is shifted toward the violet. Their reverse behavior in the

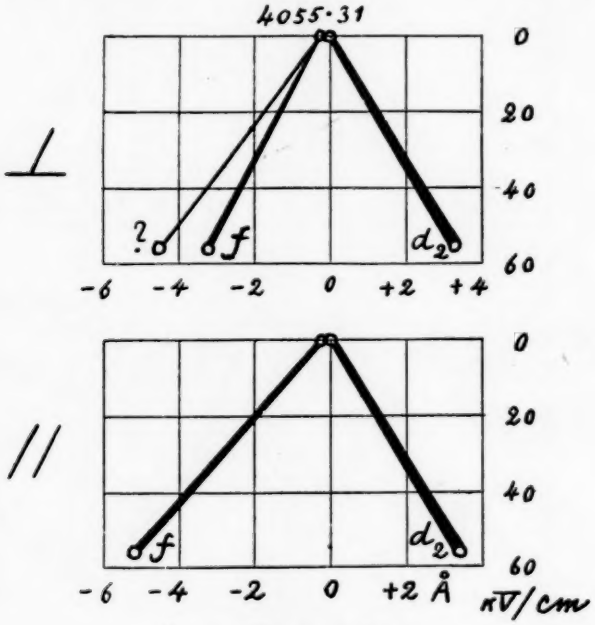


FIG. 2.—Silver $2p_2$–4d group

electric field may possibly be accounted for by supposing that, in combining with $2p_1$ or $2p_2$, the doublet character of the 4f-term becomes effective and that $4f_1$ and $4f_2$ behave oppositely in the electric field, just as in the case of $4d_1$ and $4d_2$. The double branches of $2p_2-4f$ for the *n*-components observed by Takamine may, as in $2p_1-4d_1$, be due to contamination of the strong *p*-component.

The new line 4078.8 A, measured to be 4081.7 and classified as of "Koch's type" by Takamine, is undoubtedly $2p_2-4p_2$, the calculated wave-length of which is 4079.13. The combination $2p_2-4p_1$ with the calculated wave-length 4066.25 is not observed.

$2p_1-5d$ (λ 3811) group.—$2p_1-5p_2$ and $2p_1-5p_1$ appear both as *p*- and *n*-components, and are shifted toward the red in the field. $2p_1-5d_1$ resolves into three lines, all displaced toward the violet. These three lines could be designated as $2p_1-5d_1$, $2p_1-5f$, and $2p_1-$

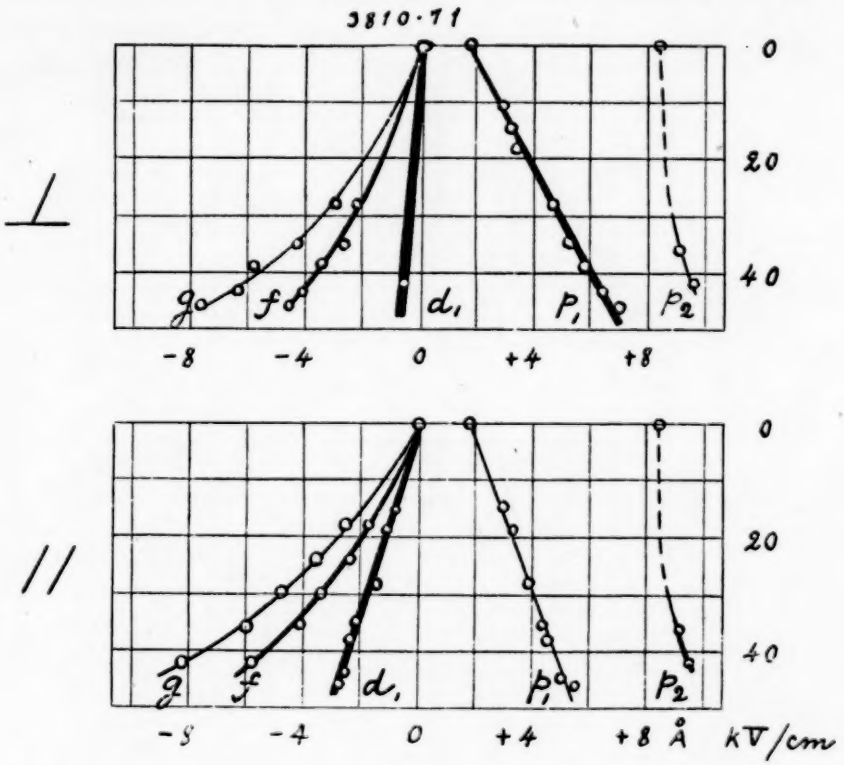


FIG. 3.—Silver $2p_1$–5d group

5g, beginning with the least refrangible one (Fig. 3). Under higher resolving-power, $2p_1-5f$ intersects with $2p_1-5d_1$ in the electric field as shown by Takamine. It is a remarkable fact that an exactly similar behavior is shown by the corresponding series lines of helium; e.g., the orthohelium line $2p-5f$ intersects with $2p-5d$ (λ 4026).[1] Furthermore, what Foster calls the "vanishing components" about $2p-5d$ of orthohelium are also observed in the case of silver. For

[1] J. S. Foster, *Physical Review*, **23**, 667, 1924.

instance, the p-components of $2p_1-6d_1$ and $2p_1-7d_1$ diminish in intensity, the latter almost disappearing, in increasing electric fields.

A line, expected to be $2p_1-5d_2$ (λ O. = 3811.72 A, λ C. = 3811.54), appears on the red side of $2p_1-5d_1$, and disappears immediately as the field increases.

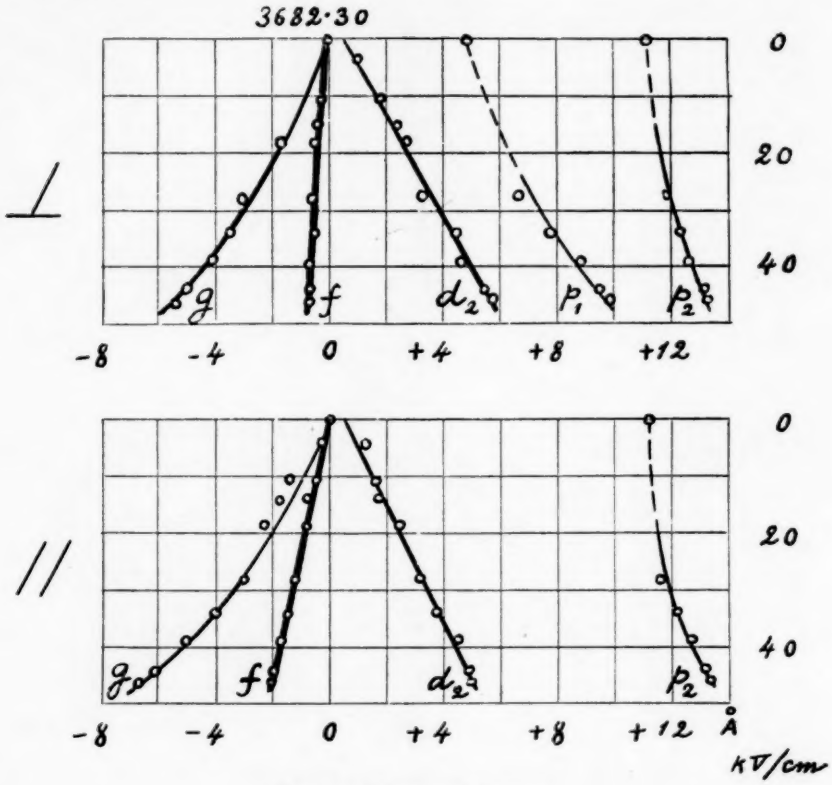


FIG. 4.—Silver $2p_2$–5d group

$2p_2-5d$ (λ 3682) group.—$2p_2-5p_2$ appears with both p- and n-components, while $2p_2-5p_1$ has only a normally polarized part. The main group is composed of three lines; one displaced toward the red and the others toward the violet. We may safely assign them to $2p_2-5d_2$, $2p_2-5f$, and $2p_2-5g$, beginning with the least refrangible one (Fig. 4). The first one ($2p_2-5d_2$) starts from a slightly separated position on the red side of the others. Even this anomalous separation finds its counterpart in the orthohelium spectrum.

$2p_1-6d$ (λ 3624) group.—One of the isolated new lines ($2p_1-6p_2$) appears only in the *n*-pattern (Fig. 5). In the main group we observe four lines of *n*- and three of *p*-polarization, converging to $2p_1-6d$ in the zero field. The four lines may be denoted as $2p_1-6d_1$, $2p_1-6f$, $2p_1-6g$, and $2p_1-6h$, respectively, one of them (probably $2p_1-6f$) being too faint in the *p*-pattern to be observed.

$2p_2-6d$ (λ 3507) group.—$2p_2-6p_1$ appears only in the *n*-pattern; $2p_2-6p_2$ in both *p*- and *n*-patterns. $2p_2-6d_2$ resolves into

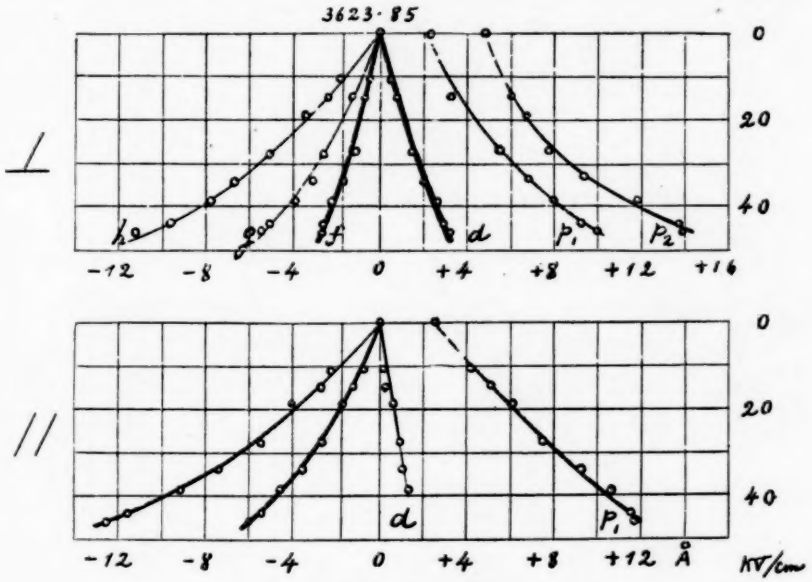


FIG. 5.—Silver $2p_1-6d$ group

three lines, which we assign to $2p_2-6d_2$, $2p_2-6f$, and $2p_2-6g$, respectively.

$3p_1-7d$ (λ 3520) group.—Already too faint to allow an exact measurement. In *n*-pattern $2p_1-7d_1$ resolves into four lines which may be assigned to $2p_1-7d_1$, $2p_1-7f$, $2p_1-7g$, and $2p_1-7h$. The first one of these four shows no *p*-component. This line, as well as $2p_1-6d_1$, corresponds to the so-called "vanishing components" of orthohelium.

In Table I we give the observed and calculated wave-lengths of all the lines described above, together with the magnitudes of the

electric effect upon them in a field given in the last column (estimated from the effect on the Balmer lines). *i* indicates the relative intensity in the group.

TABLE I

STARK EFFECT FOR SILVER LINES

SERIES	λ O. I.A.	λ C. I.A.	*n*-COMP. Δλ A	*n*-COMP. *i*	*p*-COMP. Δλ A	*p*-COMP. *i*	E 10^4 V/CM
$2p_2$ — $4p_2$	.....	4238.39	.....		.....		
$4p_1$	4226.9	4224.43	+ 1.2	1	+ 0.9	1.5	4.3
$4d_2$	4212.60*	———	+ 1.6	9	+ 1.3	6	4.3
4f	4212.3	4212.68					
$4d_1$	4210.71*	———	− 2.2	10	− 3.7	8	4.3
$2p_2$ — $4p_2$	4078.8	4079.13	+ 0.2	1	+ 0.2	0.7	4.3
$4p_1$	.....	4066.25	.....		.....		
$4d_2$	4055.31*	———	+ 2.0	7	+ 1.8	5	4.3
4f	4055.1	4055.36	− 3.1	7	− 3.9	7	4.3
$2p_1$ — $5p_2$	3819.1	3821.17	+ 0.6	1.5	+ 2.0	1	4.0
$5p_1$	3813.38	3815.62	+ 2.83	2	+ 2.04	0.5	4.0
$5d_2$	3811.72	3811.54	.....		.....		
$5d_1$	3810.71*	———	− 0.35	3	− 2.80	3	4.0
5f	3810.7	3810.35	− 4.3	1	− 5.6	2	4.0
5g	3810.7	.....	− 7.7	0.5	− 8.2	1	4.0
$2p_2$ — $5p_2$	3693.5	3691.27	+ 1.5	0.5	+ 1.4	1	4.5
$5p_1$	3687.4	3686.10	+ 2.8	0.5	.....		4.5
$5d_2$	3862.30*	———	+ 5.7	2	+ 4.8	1	4.5
5f	3682.3	3681.19	− 0.74	4	− 1.78	3	4.5
5g	3682.3	.....	− 6.2	2	− 6.3	2	4.5
$2p_1$ — $6p_2$	3628.6	3629.99	+ 8.3	1	.....		4.5
$6p_1$	3626.3	3627.01	+ 7.4	1	+ 9.8	1	4.5
$6d_2$	3623.85*	———	+ 3.20	2	+ 1.00	0	4.5
6f	3623.8	3624.34	− 2.9	3	.....		4.5
6g	3623.8	.....	− 6.2	1	− 5.5	1.5	4.5
6h	3623.8	.....	−10.4	1	−11.8	1	4.5
$2p_2$ — $6p_2$	3513	3512.57	+ 5	0.5	+ 5.4	0.5	4.7
$6p_1$	3511	3509.91	+ 4	0.5	.....		4.7
$6d_2$	3507.35*	———	+ 1.66	0.5	+ 1.12	1	4.7
6f	3507	3507.24	− 2.7	2	− 4.2	1.5	4.7
6g	3507	.....	− 7.6	0.5	− 8.7	0.5	4.7
6h	3507	.....	.....		.....		
$2p_1$ — $7p_2$	.....	3524.52	.....	0.5	.....		4.7
$7p_1$	3522.4	3523.46	.....	0.5	.....	0.5	4.7
$7d_1$	3520.35*	———	+ 6.7	1	.....	0	4.7
7f	3520	3521.42	0	2	− 1.8	0.5	4.7
7g	3520	.....	− 6.1	1	− 8.0	0.5	4.7
7h	3520	.....	−12.5	0.5	−12.6	0.5	4.7

* Wave-lengths given in Fowler's *Report*. Other lines were measured relatively to these lines.

Assuming the term shift to be approximately proportional to the electric field and the final levels $2p_1$ and $2p_2$ to be not affected or very little so by the field, we give values of the silver terms at

TABLE II

SILVER TERMS (CM^{-1})

	Undisturbed ν (Mean of Values Derived from $2p_1$- and $2p_2$-Combinations)	$\Delta\nu$ in 20,000 V/cm			
		$2p_1$-Combinations		$2p_2$-Combinations	
		Normal	Parallel	Normal	Parallel
$4p_2$	7034	.....	.....	+ 0.6	+ 0.6
$4p_1$	6972	+ 3	+ 2	.....	.....
$4d_2$	6891.4	+ 4	+ 3	+ 6	+ 5
$4d_1$	6880.7	− 6	− 9	.....	.....
4f	6890.3	+ 4	+ 3	− 9	−11
Hydrogen	6854.9				
$5p_2$	4457	+ 4	+14	+ 5	+ 5
$5p_1$	4420	+10	+ 7	+ 9	.....
$5d_2$	4394.3	.....	.....	+20	+17
$5d_1$	4388.6	− 1.2	−10	.....	.....
5f	4388	−15	−19	− 3	− 7
5g	4388	−26	−28	−22	−22
Hydrogen	4387.1				
$6p_2$	3079	+29	.....	+17	+19
$6p_1$	3062	+25	+33	+15	.....
$6d_2$	3040.2	.....	.....	+ 5.8	+ 3.9
$6d_1$	3035.9	+10	+ 3.2	.....	.....
6f	3035	− 3	.....	−11	−18
6g	3035	−20	−18	−32	−37
6h	3035	−33	−38	.....	.....
Hydrogen	3046.6				
$7p_1$	2241	.....	.....		
$7d_1$	2224.8	+23	.....		
7f	2222	0	− 6.3		
7g	2222	−21	−28		
7h	2222	−44	−44		
Hydrogen	2238.3				

zero field (or more correctly, in the least-disturbed conditions which allow the measurement of the corresponding lines) and at about 20,000 V/cm in Table II.

We may summarize the general character of the above-described normal and forbidden series lines in an electric field as follows:

$2p_1 - mp_1$ and $2p_2 - mp_2$.—Easily excited in a weak field; the

higher the term number, the easier becomes the excitation. All lines are displaced toward the red, and the displacement increases with the term number.

$2p_1 - mp_2$.—More difficult to be excited than $2p_1 - mp_1$. Generally, the *n*-components are stronger than the *p*-components and the polarization becomes more complete as the term number increases. All lines displaced toward the red.

$2p_2 - mp_1$.—Displacements and polarization are quite the same as for $2p_1 - mp_2$.

TABLE III

STARK EFFECT FOR COPPER LINES

SERIES	λ O. I.A.	λ C. I.A.	*n*-COMP. Δλ A	*n*-COMP. *i*	*p*-COMP. Δλ A	*p*-COMP. *i*	*E* 10^4 V/CM
$2p_1-$ 4d	4063*	———	+0.90	10	+0.90	10	5.1
$2p_1-$ 4f	4056.5	4056.53	− .90	3	−0.90	3	5.1
$2p_2-$ 4d_2	4022.67*	———	+0.71	8	+0.80	8	5.1
$2p_2-$ 4f	4015.7	4016.09	−1.24	2	−0.77	2	5.1
$2p_1-$ 5d	3687.5*	———	+3.0	3	+5.6	2	5.1
$2p_1-$ 5f	3686.0	3685.67	+0.2	3	+0.4	3	5.1
$2p_1-$ 5g	3679.7	.....	.....		−2.9	2	5.1
$2p_2-$ 5d_2	3654.3	———	+2.8	2	+4.8	1	5.1
$2p_2-$ 5f	3652.1	3652.23	0.0	3	0.0	2	5.1
$2p_2-$ 5g	3649	.....	.....		−2.7	1	4.2
$2p_1-$ 6d	3512.0	3512.15	+7.6	1	+8.7	1	4.2
$2p_1-$ 6f	3511.8	3511.67	+1.3	1	+1.4	0.5	4.2
$2p_1-$ 6g	3510.0	.....	−2.1	0.5	.....		.....

[1] Y. Fujioka, *loc. cit.*

$2p_1 - md_1$.—m = 4 and m = 5 are displaced toward the violet, but all the higher members are toward the red and tend to become "vanishing lines."

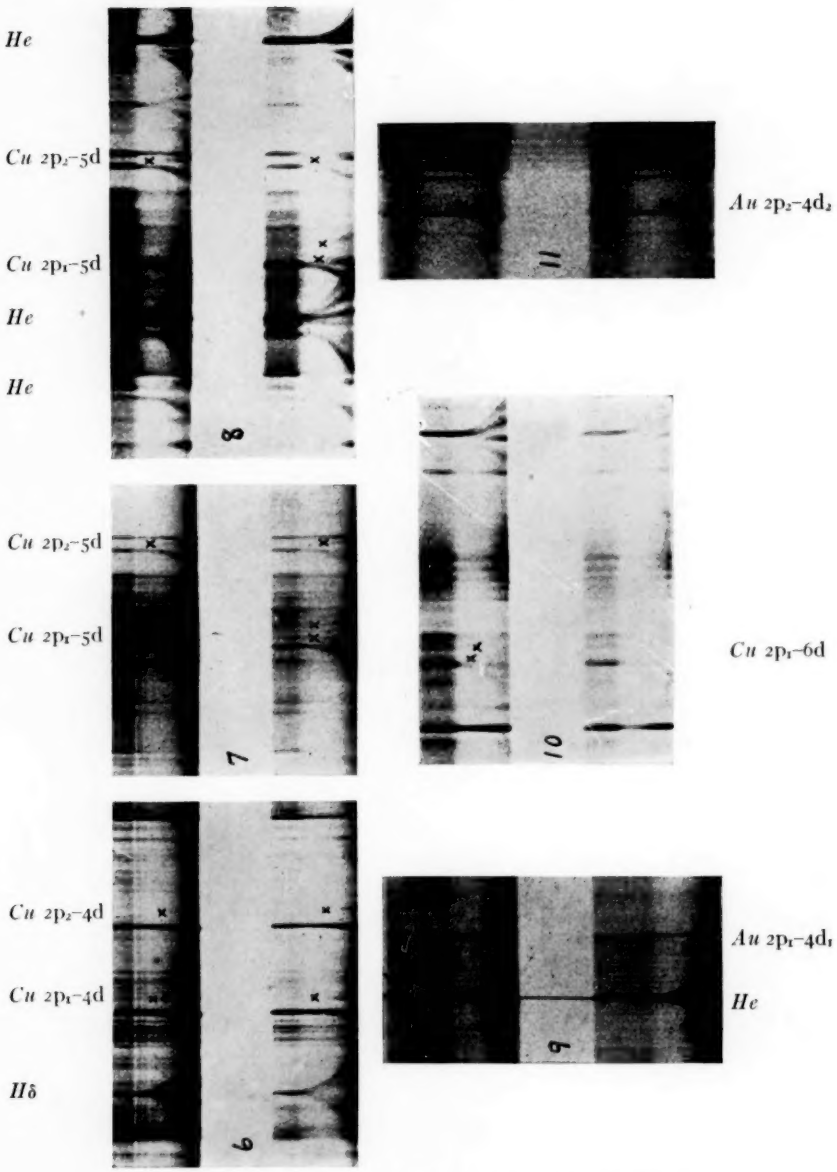
$2p_1 - md_2$.—Of this series only m = 4 and m = 5 were observed by Takamine. He showed that the first one is shifted toward the red and both are weakened in the electric field.

$2p_2 - md_2$.—Seem to be displaced toward the red. Do not become "vanishing components."

$2p_2 - md_1$.—No line of this combination, violating the inner quantum-number rule, has been observed.

2p − mf and other forbidden combination lines which violate the

PLATE V



STARK EFFECT FOR THE SPECTRA OF COPPER AND GOLD

6, 7 – $E_{max}=4.9\times10^{4}$V/cm in chlorine; 8 – $E_{max}=4.2\times10^{4}$V/cm; 9 – $E_{max}=4.7\times10^{4}$V/cm; 10 – $E_{max}=4.2\times10^{4}$V/cm; 11 – $E_{max}=4.6\times10^{4}$V/cm.

azimuthal quantum-number rule by more than two units almost coincide with 2p−md in a zero field. mf, mg, mh, etc., are very close to md even for small values of m. They can be separately measured only under perturbed conditions.

With respect to other series than those described above, Takamine observed a slight shift of 2p−5s toward the red, but in the present investigation no such effect could be detected.

TABLE IV

COPPER TERMS (CM $^{-1}$)

UNDISTURBED		TERM-SHIFT IN A FIELD ABOUT 20,000 V/CM			
		$2p_1$-Combinations		$2p_3$-Combinations	
		n-Comp.	*p*-Comp.	*n*-Comp.	*p*-Comp.
$4d_2$	6920.8	+ 2	+ 2	+ 2	+ 2
$4d_1$	6917.1	+ 2	+ 2	.....	.....
4f	6880.0	− 2	− 2	− 3	− 2
Hydrogen	6854.9				
$5d_2$	4415.5	.....	.....	+8	+14
$5d_1$	4413.4	+ 9	+16	.....	.....
5f	4400.0	+ 0.6	+ 1.1	0	0
5g	4352	.....	− 8	.....	−8
Hydrogen	4387.1				
$6d_2$	3062.0	.....	.....	.....	.....
$6d_1$	3059.9	+29	+34	.....	.....
6f	3056.0	+ 5	+ 5	.....	.....
6g	3041	− 8	.....	.....	.....
Hydrogen	3046.6				

III. STARK EFFECT FOR COPPER LINES

This has already been observed for the 2p−md series by Takamine up to the term m = 5. An extension to this work is made and certain forbidden series are found in the present investigation. The behavior of the 2p−md series line in electric fields is much simpler than that of the silver lines; it resembles more closely that of zinc cadmium, and mercury.[1]

It is a remarkable fact that in the case of copper none of the 2p−mp combinations appear. New lines on the violet side of the diffuse series lines can be assigned with certainty to 2p−mf and 2p−mg combinations. The 2p−md lines are generally displaced toward the

[1] Y. Fujioka, *loc. cit.*

red, the displacement increasing with term numbers. $2p_1-4d_1$ and $2p_1-4d_2$ are not resolved on our plates, but in Takamine's experiment they were resolved and both displaced toward the red.

$2p_{1,2}-mf$.—m=4 are displaced toward the violet, while m=5 and 6 are shifted toward the red. Polarization is not clearly observed for all these lines.

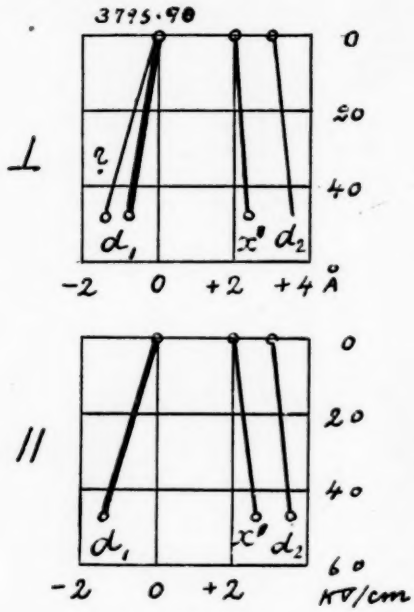


FIG. 6.—Gold $2p_1$–4d group

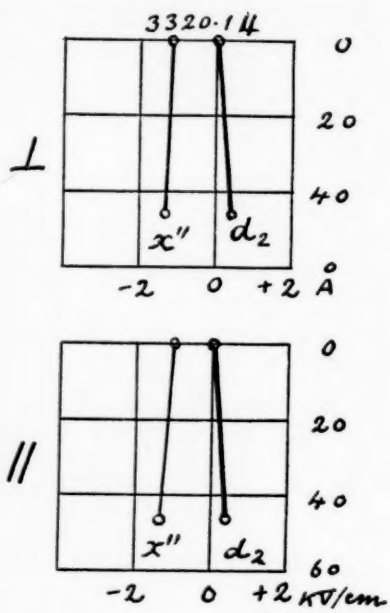


FIG. 7.—Gold $2p_2$–4d group

IV. STARK EFFECT FOR GOLD LINES

Series in the arc spectrum of gold have recently been investigated by McLennan and McLay.[1] The first two members of the diffuse series of the doublet system are:

$2p_1-3d_2$.....4811.61 A	$2p_1-4d_2$.....3801.97 A
$2p_1-3d_1$.....4792.60	$2p_1-4d_1$.....3795.90
$2p_2-3d_2$.....4065.08	$2p_2-4d_2$.....3320.14

No appreciable electric effect on the first set could be observed on our plates, while for the second set the effect was quite clear. A new line appears between $2p_1-4d_1$ and $2p_1-4d_2$, and is shifted toward the red in increasing electric fields (Fig. 6). Another new

[1] *Proceedings of the Royal Society*, A, **112**, 95, 1926.

line, separated by 2.7 A on the violet side of $2p_2-4d_2$, is shifted toward the violet (Fig. 7). We call these lines $2p_1-x'$ and $2p_2-x''$. Using McLennan's term values, $2p_1=33286.8$, $2p_2=37102.4$, we find $x'=6977$, $x''=6967$, in the least-disturbed conditions accessible to measurement. The modes of appearance of these lines resemble $2p_{1,2}-4f$ of silver remarkably well. To complete the analogy, x' increases and x'' decreases with increasing electric fields, as we see in Table VI.

TABLE V

STARK EFFECT FOR GOLD LINES

SERIES	λ O. I.A.	*n*-COMP.		*p*-COMP.		E 10^4 V/CM
		Δλ A	*i*	Δλ A	*i*	
$2p_1-4d_2$.....	3801.97*	.....	0	+1.0	1	4.7
$2p_1-x'$.....	3799.8	+0.6	1	+1.1	1	4.7
$2p_1-4d_1$.....	3795.90*	−1.8 −2.7†	2 0.5†	−2.8	2	4.7
$2p_2-4d_2$.....	3320.14*	+0.9	2	+0.8	2	4.6
$2p_2-4x''$.....	3317.5	−0.3	1	−0.5	1	4.6

* Taken from McLennan and McLay's paper.
† Contamination of *p*-component?

TABLE VI

GOLD TERMS (CM^{-1})

UNDISTURBED		Δν IN 20,000 V/CM FIELD			
		$2p_1$-Combinations		$2p_2$-Combinations	
		n-Comp.	*p*-Comp.	*n*-Comp.	*p*-Comp.
$4d_1$.....	6950.1	−5.3	−8.2	———	———
$4f\{x''$.....	6967	———	———	−1.2	−2.0
$4f\{x'$.....	6977	1.8	+3.2	———	———
$4d_2$.....	6991.9	.....	+2.9	+3.5	+3.2

We may assign to them the doublet 4f-term, although we have too few data to verify this assignment.

In conclusion, the authors wish to express their most sincere thanks to Professor Takamine for his valuable suggestions and kind guidance; also to Dr. O. Laporte, of the University of Michigan, who very kindly read and corrected the manuscript.

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THE EXCITATION OF THE SPECTRA OF CARBON MONOXIDE BY ELECTRONIC IMPACTS

By O. S. DUFFENDACK AND G. W. FOX

ABSTRACT

The *ionization potential* of the carbon-monoxide molecule was *determined* with the *low-voltage arc* to be 14.3 volts.

The band spectra of carbon monoxide were excited in tubes with two and three electrodes, and the *excitation potentials* for the various systems *were determined* by the photographic method. *Two parallel* sets of *levels* occur for the *neutral* molecule with values 6.0, 10.34, 11.35 volts for one set and 8.0 and 10.73 volts for the other set. The systems have been correlated to transitions between these levels.

The third positive carbon bands reported by Deslandres have been shown to be due to carbon monoxide.

Complete analysis of the *third* positive system has been made, and a new system proposed.

The *ionized* carbon-monoxide molecule was found to have *three levels* with values 16.9, 20.0, and 22.9 volts. The systems of *negative bands* have been correlated to *transitions* between these levels.

This paper deals with the critical potentials and spectra of carbon monoxide. It is companion to papers by O. S. Duffendack,[1] D. C. Duncan,[2] and L. L. Lockrow[3] in which have been described researches on hydrogen, nitrogen, and oxygen. The work is divided into three parts. The first deals with the electrical behavior of the gas under various conditions of electronic excitation and pressure, and from these data the ionization potential is determined. The second part deals with the determination of the critical voltages necessary to excite the band spectra of neutral and ionized carbon monoxide. The last part deals with the origin of the "carbon-oxide" bands. (In the last two parts of the work the spectroscope was used.)

PART I. THE ARC

The low-voltage arc provides an ideal means for the production of the spectra of gases, and also furnishes a very easily controlled arrangement for examining their electrical characteristics. This method of excitation was used throughout the entire work. The

[1] *Physical Review*, **20**, 665, 1922.

[2] *Astrophysical Journal*, **62**, 145, 1925.

[3] *Ibid.*, **63**, 205, 1926.

theory of the low-voltage arc as developed by Duffendack[1] and by K. T. Compton[2] leads to the conclusion that the sudden large increase in current results from a neutralization of the negative space-charge surrounding the filament. At low voltages, i.e., at low electronic velocities, the region immediately surrounding the filament will be at about the potential of the filament, provided the temperature of the filament is high enough to insure a copious emission. The current is limited by the space-charge. At higher voltages some ionization may set in due to the increased energy of the electrons at collision. The positive ions formed neutralize the negative space-charge surrounding the filament; and, since each ion can neutralize the space-charge of a large number of electrons, a large increase in electronic emission results; this is termed "striking the arc." Under this condition practically the entire fall of potential across the tube occurs in the immediate vicinity of the cathode. As a result there is increased ionization near the cathode, with the consequent formation of more positive ions which still more fully neutralize the space-charge of the electrons. The result is a value of the current which closely approximates that given by O. W. Richardson's equation.[3]

APPARATUS

The discharge tube was made of pyrex glass of the form shown in Figure 1. Each end was closed by a ground-joint stopper, one of which carried the filament leads of tungsten, and the other the lead to the anode and a second electrode which supported a grid in a later part of the investigation. On one side of the tube was a quartz window through which the radiation could be photographed. From the top of the tube a glass connection led to the liquid-air trap and then to the pumping system. The ground joints of the stoppers and window were sealed with hard De Khotinsky cement, water cooled, and no stopcock grease was used. The tube and electrodes were completely outgassed before *CO* was admitted.

A tungsten anode and filaments in the form of a five-turn helix of 12-mil tungsten wire were used throughout most of the work, the filaments being spot-welded to the supports. The anode was made

[1] *Physical Review*, **20**, 665, 1922.

[2] *Ibid.*, **21**, 266, 1923.

[3] *Emission of Electricity from Hot Bodies*. 2nd ed. p. 35.

in the form of a square U turned on its side. Both filament and anode were so arranged as to lie on the axis of the tube, the distance from the filament to any part of the anode being about 6 mm. By this arrangement the characteristic "sky-blue" glow of carbon monoxide could be well concentrated between the electrodes. A single-stage mercury pump backed by a Cenco Hyvac pump made up the vacuum system. Pressures were read on a McLeod gauge.

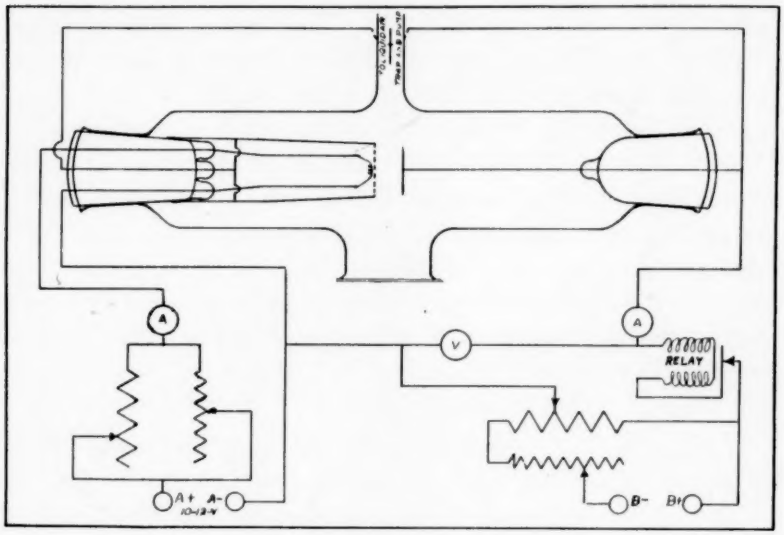


FIG. 1.—Discharge tube and circuit arrangement

The carbon-monoxide gas was generated outside the system entirely. Especially prepared oxalic-acid crystals were heated in the presence of concentrated sulphuric acid. Carbon monoxide containing a trace of carbon dioxide is produced by this method. The two gases, together with any water-vapor not removed by the acid, were then passed through a wash bottle containing a concentrated solution of potassium hydroxide to remove the CO_2. The remaining gas was then passed through a drying-tube containing phosphorus pentoxide to remove still further any moisture, and then into an evacuated flask, which was allowed to fill up to atmospheric pressure. This flask of gas was then sealed to the vacuum system and isolated from the discharge tube. Since in this type of work the gas

pressure never exceeds a few mm at most, two bottles of gas were found to more than suffice for the entire investigation.

The circuit employed was the usual low-voltage arc circuit. The plate current was read on a Paul unipivot milliammeter. The plate voltmeter was connected from the plate of the tube to the negative side of the filament. Since the middle turn of the filament is the hottest and therefore the most efficient in electronic emission, one-half the voltage drop along the filament has been, in all cases, subtracted from the voltmeter reading, and this corrected value is called the true difference of potential between the electrodes. Further dicussion of this correction will be made later.

DETERMINATION OF THE ARCING POTENTIAL

To determine the effect of pressure changes on the arc, a series of runs were made at constant filament voltage and various pressures. In all cases the potential of the "break" was somewhat lower than that of the "strike" and is believed to be more characteristic of the ionizing potential.[1] It is this voltage from which the ionizing potential was determined. As it is a well-known fact, borne out by repeated observation, that an arc cannot be maintained in a diatomic gas at a voltage less than the ionization potential, this procedure seems justifiable. At low pressures the value of the mean free path between effective collisions is large compared to the interelectrode distance. Little ionization occurs until the part of the path in which ionizing collisions can occur is a large fraction of the total path. At the optimum pressure the electrons make the largest number of ionizing collisions while traveling from cathode to anode. Above this pressure, electronic energy is lost by impacts and momentum exchanges with gas molecules. Over the range from 0.4 mm to 1 mm no particular difference could be noted in the breaking-potential of the arc. Above or below this range, however, the breaking-potential increased markedly. The lowest voltage at which the arc could be maintained in carbon monoxide was 14.3 volts, which value represents the ionization potential of the molecule, in agreement with the values obtained by other methods.[2] Knowing this

[1] Duffendack, *op. cit.*

[2] *Bulletin of the National Research Council on Critical Potentials*, No. 48, p. 120.

value, it can be said with certainty which of the emitted radiations are due to the neutral, and which are due to the ionized, molecule. This at once divides the field of investigation into two parts which will be dealt with in detail later.

There were no oscillations observed in carbon-monoxide arcs. A cathode-ray oscillograph was connected across the discharge tube, and typical runs were made in exactly the same manner as when determining the ionizing potential. No fluctuation in the cathode beam across the field could be detected except the slight fall or rise of the beam corresponding to the slight voltage drop or rise indicated also by the voltmeter when the arc struck or broke.

PART II. THE CARBON-MONOXIDE SPECTRA

Few gases possess so diversified a band spectrum as carbon monoxide. From well out in the red its band systems extend through the visible and ultra-violet regions apparently as far as one chooses to penetrate. In this part of the work the object has been to isolate these different spectra, so far as possible, and to determine the critical amounts of energy necessary to excite them. In this way one can assign each particular system to its proper origin and establish some physical relations between systems. With this object in view, therefore, a spectrographic study was made of the radiations emitted by the carbon-monoxide molecule when excited by electrons of known velocities.

Above the ionization potential, low-voltage discharges were maintained in carbon monoxide contained in the simple two-element tube previously mentioned. For investigation below the ionization potential, this tube was modified by the introduction of a third electrode: a grid made of a cylinder of thin nickel having a small platinum gauze window in the end. This cylinder fitted over the filament supports and was mounted so that the distance from filament to gauze window was less than one millimeter. The plate and grid were then electrically connected, and radiation from the force-free region was photographed through the quartz window in the side of the tube (Fig. 1).

Observations obtained above the ionization potential with this arrangement and with the two electrode tubes were in very good

agreement. This is to be expected according to the theory of the low-voltage arc, for when the arc strikes, a redistribution of potential takes place in the tube, so that almost the entire voltage drop occurs very close to the cathode. We should expect, therefore, that on the first collision the electrons should have a velocity practically equal to the applied potential difference.

The critical potentials for each system were determined by photographing throughout a certain voltage range, first using volt-intervals, and then, in cases where greater accuracy could be had, by using intervals as small as 0.1 volt. For systems which are not superposed, accuracy as great as 0.2 volt is entirely possible. If conditions could be arranged to allow the appearance of a single system at a time, accuracy as high as 0.2 volt should be the rule. Unfortunately, in those systems above the ionizing-potential, one system may come in after several others in the same region are developed; and it is difficult, in such cases, to locate the excitation potential closer than half a volt.

The values of the critical potentials listed in Table I are *corrected* values. The potential difference was in all cases applied between the plate and the negative end of the filament. As this voltage is increased, the potential difference between the anode and the negative end of the filament reaches the critical value first. Consequently, the few electrons emitted by this portion of the filament attain velocities of such magnitude that on long exposure a faint spectrogram of the band system—or at least, of the bands originating in the lower vibrational levels—can be developed. As the applied voltage is increased, a point is reached at which the potential difference between the central turns of the helical filament and the plate reaches the critical value. The central turns of the filament are the hottest and furnish nearly all the electronic emission. At this point the band system suddenly develops. A further slight increase in the applied voltage brings the potential difference between the last portion of the filament and the anode up to the critical value. This will slightly increase the number of critical-speed electrons, but the percentage increase is so small that no effect on the intensity of the band systems can be detected. The values of the critical potentials are, therefore, the values of the voltage between the anode

and the middle of the heated cathode after the band system has developed. This means that half the IR drop along the filament has been in each case subtracted from the actual voltmeter reading.

The situation is shown graphically in Figure 2, which is based on a calculation from a typical 5-turn tungsten filament as used throughout this work. The information regarding tungsten emission used in sketching the curve was taken from data furnished by the General Electric Company and the Nela Research Laboratory. Some allowance was made for possible heating of adjacent turns due to radiation, but in a personal letter Dr. Dushman says this effect

TABLE I

CRITICAL POTENTIALS

Band System	Direction of Shading	Excitation Potential Observed (Volts)
Ångström (second positive)	Toward violet	10.7
Deslandres (fourth positive)	Toward red (beyond range of investigator)	8.0 (calc.)
Cameron	Toward red (not observed)	6.0 (calc.)
Third positive	Toward violet	10.2
3 A (positive)	Toward violet	11.1
Deslandres (first negative)	Toward red	20.0
"Comet-tail" (third negative)	Toward red	16.9
Baldet-Johnson	Toward violet	22.9

is small for filaments of the type used. This points at once to the middle turn as the source of most of the electronic emission.

To test the validity of the correction, determinations were made of the critical potential of a certain system using both the tungsten filaments and filaments of oxide-coated platinum. In these two cases the voltage drops along the filaments were widely different: for the tungsten, about 2.5 volts, and for the oxide filaments, 0.8 volts. The correction for initial velocity of electrons should also be very different since the oxide filament was run at a very dull-red heat and the tungsten at a white heat. Further, there are different contact electromotive forces to consider. However, determinations of the excitation potential in the two cases, applying the same correction, i.e., one-half the voltage drop along the filament, gave values of the critical potential only 0.1 volt apart. Since determina-

tions of the critical potentials with two different types of filaments agree within 0.1 volt, when the same correction is applied in each case it would seem that this method of correction is legitimate.

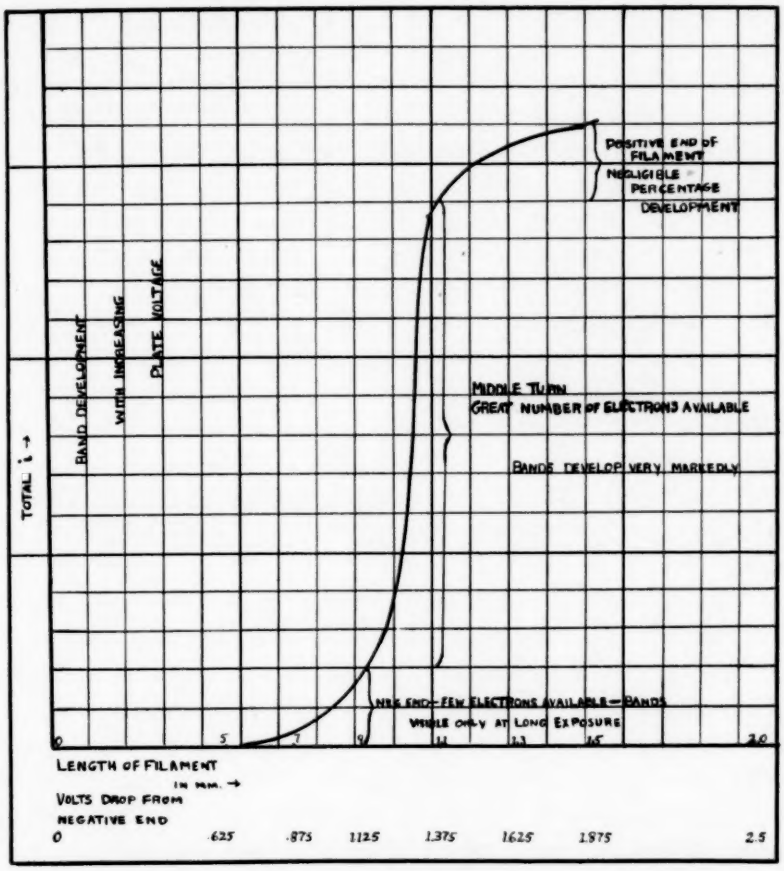


FIG. 2.—Curve showing band development with increasing voltage

To further test this point, a 3-turn tungsten spiral was used to check the determined excitation potential of one system. The voltage drop over this filament was less than over the 5-turn spiral, and emission is almost wholly from the middle turn. Again the same method of correction led to agreement in results. It may be said, furthermore, that the corrections applied in all cases were such as

to make the excitation potential of the Ångström system 10.7 volts. The excitation potential of this system can be computed with certainty because of its relation to the fourth positive system which has been obtained in absorption.

RESULTS

The carbon-monoxide band spectrum comprises three negative systems commonly known as the "first negative," "third negative" or "comet-tail" bands, and the "Baldet-Johnson" bands. The positive systems are known as the "second positive" or "Ångström" bands, the "Cameron" bands, the "third positive" bands, and a system to be designated as the "3 A" bands. These systems, together with their directions of shading and corresponding observed excitation potentials, are given in Table I.

The Ångström bands.—In an old work[1] published in 1875, Ångström and Thalén gave the measurements of the heads of this band system which has ever since borne Ångström's name. These bands begin at 6622 A and extend intermittently throughout the entire visible spectrum. They do not occur in groups, but as single bands having a very decided edge and shading off markedly toward the violet.

This system was not excited until the P.D. between the anode and middle of the filament reached 10.2 volts. By long exposure at this voltage, the edges at 4511 A and 4834 A could be seen very faintly. Increasing the voltage in 0.1-volt steps brought out a decided development of the system at 10.7 volts, above which value no great development occurred. Consequently the excitation potential was fixed at 10.7 volts in accordance with the expected development of the bands with increasing voltage as outlined above. The same procedure was followed for each band system. It is interesting to note that although this system comes in well below the ionizing-potential it is exceedingly strong no matter what voltage is applied above its exciting potential. At 200 volts these bands are remarkably brilliant, their fine structure being especially well developed.

The Ångström bands are best produced in the low-voltage arc

[1] *Nova Acta Regiae Societatis Scientiarum Upsaliensis*, **9**, 1875.

at pressures of the order of a millimeter. At low pressures of 0.01 mm, or less, nothing but the edges remain, and even these are faint. This is a high-pressure system.

Table II gives the wave-lengths and wave-numbers of these bands. They are shown in two groups, "A" and "B," as previously classified.

TABLE II

Ångström Bands

Previously Reported λ	By Writer λ	1/λ	Intensity
	Group A		
4209.0	Not observed	23759	.....
4511.0	4512.1	22156	10
4834.0	4834.8	20678	9
5197.0	5197.6	19235	10
5607.5	5609.8	17821	10
6078.0	6080.2	16442	8
6622.0	6619.9	15102	5
	Group B		
4131.0	4126.4	24227	7
4394.0	4392.2	22756	8
4697.0	Not observed	.....	.....
5015.0	Not observed	.....	.....
5397.5	Not observed	.....	.....
5817.0	Not observed	.....	.....
	New Bands		
	4380.1	22824	7
	3894.8	25667	6
	3679.5	27170	9

As will be noticed, only part of the bands originally recorded as belonging to this system have been observed during this investigation. The bands at 4380.1 A, 3894.8 A, and 3679.5 A have not been reported previously. These bands are very strong on all plates of the Ångström system; and, although they do not fit into the scheme proposed by Raymond T. Birge,[1] their appearance and behavior indicate that they should be included in the system. His arrange-

[1] *Nature*, **117**, 230, 1926.

ment of the so-called "B" group[1] does not fit the given wave-lengths very well. Neither do the newly observed bands fit into this arrangement. Indeed it seems doubtful if his analysis is a legitimate one, since the existence of so many of the reported edges is questionable. The bands at 4380 A and 4394 A show curious intensity relations with the voltage. At low voltages of the order of 15 volts, 4393 A is very much the stronger; while at higher voltages, of around 100, 4394 A practically disappears and 4380 A is very strong. Further analysis will be necessary to straighten out this matter over which there has been some controversy.[2,3] Later, a suggestion will be offered to account for the bands not observed in this investigation. Plate VI shows the development of the Ångström system through the critical voltage range.

Fourth positive system.—This system originally classified by Deslandres is another familiar one commonly attributed to carbon. It usually appears in Geissler-tube discharges where the tubes have been sealed with stopcock grease. There are in all about 150 bands in this system, extending from about 2500 A down to about 1500 A. It was impossible to photograph any of the bands of this system below 10.5 volts. Recently, Sigmund W. Leifson, in working with absorption spectra in the extreme ultra-violet,[4] has photographed some bands which Birge[5] has identified as the 0–0, 0–1, 0–2, ... 0–11 transitions of this system. These bands lie well below the region in which the quartz spectrograph is effective, and this explains why only the bands of the higher vibrational levels could be photographed. The computed value of the critical potential as determined from the absorption spectrum is 8.0 volts. All the bands lying between 2000 A and the upper limit of the system have been identified. They are well developed above the ionizing-potential, though at higher voltages the overlapping by the first negative bands somewhat obscures them.

Cameron bands.—W. H. B. Cameron[6] recently reported a new system of bands produced by Geissler-tube discharges in neon at

[1] Birge, *Physical Review*, **28**, 1157, 1926.

[2] E. Hulthen, *Annalen der Physik*, **71**, 41, 1923.

[3] R. Mecke, *Physikalische Zeitschrift*, **26**, 217, 1925

[4] *Astrophysical Journal*, **63**, 73, 1926.

[5] *Nature, op. cit.*

[6] *Philosophical Magazine*, **1**, 405, 1926.

high pressure when carbon electrodes were used. This system lies between 2100 A and 2600 A and falls into three or four groups having five heads each. All these groups are very weak, and careful examination of a large number of plates of this region does not show them definitely present. Attempts to excite this particular system are unsuccessful, largely because of the far greater excitation of the first negative and fourth positive systems which lie in the same region. R. C. Johnson[1] definitely attributes this system to carbon monoxide and computes 6 volts as the exciting potential.

Third positive bands.—Extending from just below the visible down to about 2250 A are bands which, since their original measurement by Deslandres, have been called the "third positive carbon bands." On examination of these bands, one notices two types of structure quite dissimilar in appearance. The bands which will be called the "third positive carbon bands proper" are very complex, each band being made up of as many as five or six heavy subheads which cause them to appear as blurred and indistinct groups. Because of this complicated structure, the third positive system has been generally attributed to carbon dioxide, though no definite attempts have been made previously to determine its origin. The critical potential as observed was 10.2 volts.

In examining the development of this system, it was noted that part of it came in at almost a volt higher potential than that just given. Closer examination revealed that these latter bands, though classed with the third positive, were in reality quite different. Whereas the previously described bands were made up of as many as six heads to the group, very complicated in appearance, these consisted of only double-headed groups. Though both types were shaded toward the violet, examination showed a striking difference in the appearance of their fine structure. The bands of the double-headed nature, which will be designated as the "3 A system of carbon monoxide," have very sharp edges. Their fine structure at the higher voltages is exceedingly well defined and quite widely spaced. In general, the intensity of the 3 A system is less than that of the third positive system. Though both come in well below the ionization potential, they persist strongly at voltages above it.

The effect of reduced pressure is very noticeable on all these

[1] *Nature*, **117**, 377, 1926.

bands. At about 1 mm both systems are well developed at 13 volts exciting-potential. As the pressure is diminished, the intensity decreases markedly, until at very low pressure, when it is difficult to maintain the discharge, these systems can be photographed only with the longest exposures. This is in accord with F. Baldet's observation.[1]

Table III gives the bands which properly belong to the third positive system, together with those bands which have been named the "3 A system of carbon monoxide."

TABLE III

THIRD POSITIVE AND 3 A BANDS

Third Positive		3 A Carbon Monoxide	
λ	1/λ	λ	1/λ
2832.0	35311	2295.2	43569
2976.4	33598	2389.0	41858
3134.6	31902	2489.9	40162
3305.3	30254	2597.1	38504
3492.7	28631	2711.35	36882
3698.7	27037		
2665.5	37521		
2792.7	35805		
3079.9	32469		
3241.8	30847		
3418.2	29253		
3612.7	27680		
3825.1	26143		

In connection with the negative band systems, i.e., those due to the ionized molecule, it may be well to call attention to the effects of cumulative ionization. During the course of the work, it was found that no consistent critical potential values could be obtained from arcs maintained in the two electrode tubes unless the arc current was kept small. This effect came up first in connection with the first negative system. The higher the arc current the lower the apparent critical potential of the system became. Indeed, it was perfectly possible to excite the first negative system very strongly at the ionizing-potential with arc currents of the order of 100 milliamperes. The reason for this is that when the positive-ion concen-

[1] *Comptes Rendus*, **180**, 1201, 1925.

tration becomes large in the region surrounding the filament, the probability of exciting an ion by a second electron impact must be considered. As the energy necessary for the excitation of the ion is less than that essential for ionization, the electrons accelerated by a voltage equivalent to the ionizing-potential possess adequate energy to excite ions previously formed. However, when the ion concentration is small (small arc current) the probability of such cumulative process becomes insignificant, and both excitation and ionization must result from a single collision. In this case the electron must possess an amount of energy equal to that necessary to ionize plus that required for excitation. As mentioned previously, determinations of critical potentials were made above the ionization potential both with and without grids, the results checking very well. In such cases cumulative ionization was eliminated by keeping the arc current small.

First negative bands.—This system was discovered and classified by H. Deslandres[1] in 1903. In 1924 Johnson[2] produced the entire spectrum reported by Deslandres and was able to add several new members, chiefly on the less refrangible side of each band group. Plates produced during the present investigation show all the bands ascribed to this system by Johnson. These bands resemble the nitrogen systems in general appearance. They are arranged in distinct groups and shade toward the red. The critical exciting potential of this system as observed was 20.0 volts. Up to 200 volts, this system stands out remarkably clear and well defined. Though best developed at pressures of 0.5 to 1 mm, the first negative bands are still clearly visible at low pressures where it is difficult to maintain the discharge.

Third negative bands—"comet tails."—In 1908 A. de la Pluvinel and Baldet[3] discovered some new bands in the spectrum of the comet Morehouse which they were unable to ascribe to any of the known band systems. After the announcement of their discovery, A. Fowler[4] was able to identify the "comet-tail" bands with some he had

[1] *Comptes Rendus*, **137**, 457, 1903.

[2] *Proceedings of the Royal Society*, Series A, **108**, 343, 1925.

[3] *Astrophysical Journal*, **34**, 89, 1911.

[4] *Monthly Notices of the Royal Astronomical Society*, **70**, 176, 1909.

produced using Geissler-tube discharges in carbon monoxide at very low pressure.

The "comet-tail" bands are very readily produced in the low-voltage arc. They extend from the red through most of the visible spectrum and are characterized as narrow double-headed bands degraded toward the red. They lie in a particularly complicated region which is already occupied by the Ångström bands so that, at ordinary pressures, unless the exciting voltage is quite high, their presence might be entirely unknown. By decreasing the pressure to about 0.05 mm, the intensity of the Ångström bands can be greatly reduced and then the third negative system can be very beautifully developed. This system showed decided development at 16.9 volts, which value has been designated as the critical potential. Below this value, only by prolonged exposure could these bands be noticed at all. At higher voltage, as with the other negative systems, the "comet tails" become very brilliant, their fine structure standing out in great detail. This is a typical low-pressure system. Recently T. R. Merton and Johnson[1] have been able to develop these bands in a mixture of helium and carbon monoxide. They have measured the heads of all the bands belonging to this system with great accuracy. All the bands reported by Merton and Johnson appear on plates produced during this investigation, with good agreement as to intensities.

Baldet-Johnson bands.—This group of bands is typically a high-pressure system. Whereas the "comet-tail" bands and the first negative bands can be photographed at extremely low pressures, these entirely disappear at pressures below 0.05 mm even at voltages of the order of 100 or more. They are developed best from 0.8 to 1 mm of pressure. Baldet[2] and Johnson[3] independently discovered these bands, which are characterized as groups, having four heads each, of small wave-number separation, and degraded toward the violet. These bands were in no way visible under 22.9 volts, but at this voltage a decided development occurred, the quadruple heads of the groups standing out very clearly.

[1] *Proceedings of the Royal Society*, Series A, **103**, 383, 1923.

[2] *Comptes Rendus*, **178**, 1525, 1924.

[3] *Proceedings of the Royal Society*, Series A, **108**, 343, 1925.

SYSTEM RELATIONSHIPS

The carbon-monoxide band systems can be conveniently represented on an energy-level diagram. This scheme in general shows only the electronic transitions, but it should be borne in mind that to each electronic level belong several vibrational levels accompanied by many rotational levels.

In this scheme of levels the lowest, namely the 6 volts and 8 volts belonging respectively to the newly discovered Cameron bands and the fourth positive bands, are proposed by Johnson[1] and Birge.[2] It was impossible to determine these potentials experimentally due to the region in which these bands lie. Johnson concludes that the initial levels of the Cameron bands coincide with the final levels of the third positive bands of carbon and computes for their excitation potential 10.34 volts. The observed value is 10.2 volts, which is within the experimental error for this type of work. At 11.1 volts, the 3 A system of carbon monoxide develops. The analysis of this system leads to the conclusion that its final levels are also the initial levels of the Cameron bands. That this is a new system and not part of the third positive system is borne out by the fact that it takes an additional volt to develop it. This represents a jump of somewhat over 8000 wave-numbers, a value too large for a vibrational difference. Figure 3 shows in detail the analysis of the third positive and 3 A systems, both of which follow Deslandres progressions of the first type. The third positive system has initial vibrational states 0 and 1, with final states 0–7; and the 3 A system has only vibrational state 0, and the same final states as the companion system. It is to be noted that these systems follow the general rule observed in all band spectra, that as the excitation potential of a system approaches the ionizing-potential the number of vibrational levels diminishes. The ionization potential acts as an upper limit to the amount of energy the neutral molecule can have; and as it is approached, the probability of ionization exceeds the probability of further increase of energy in vibrational form. The fact that these two systems have entirely dissimilar appearances, so far as fine structure is concerned, is further evidence that they are two distinct band systems.

[1] *Nature*, **117**, 377, 1926. [2] *Ibid.*, p. 230.

As mentioned previously, the third positive system has very generally been attributed to CO_2 because it was believed that the

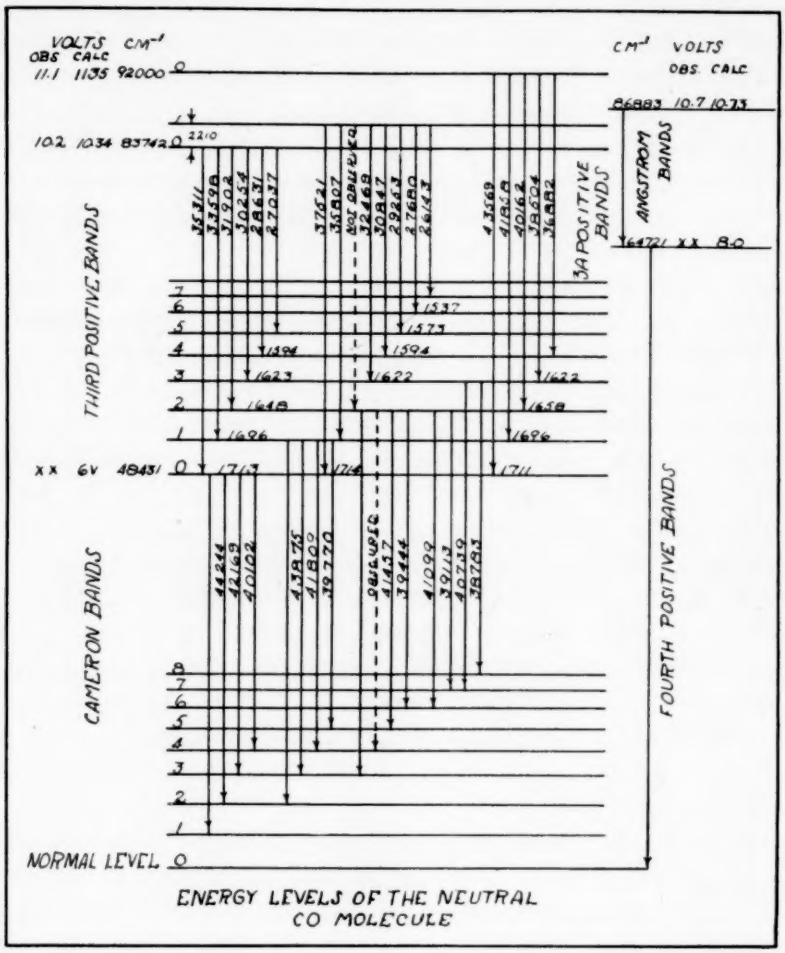


FIG. 3

bands were of too complex structure to belong to a simple diatomic molecule. However, the fine structure of these bands has never been analyzed and there is no direct evidence that they belong to CO_2. Their relation to the Cameron bands points strongly to CO as their origin as suggested by Johnson.[1]

[1] *Nature*, **117**, 377, 1926.

The initial states of the Ångström bands lie between the initial states of the third positive and 3 A groups. On examination of these bands, one is at once struck with their pronounced simplicity as compared to the systems just described. Whereas both the third positive and 3 A bands are multiple headed, the Ångström bands possess very sharp single heads and degrade rapidly toward the violet. In their simplicity they resemble quite markedly the bands of the fourth positive system. Birge[1] has recently shown that the initial states of the fourth positive bands are identical with the final states of the Ångström system. His computed excitation potential almost exactly agrees with the value observed for this system. If 8 volts is accepted as the exciting-potential of the fourth positive system and if 4512 A is the 0–0 band of the Ångström system, as Birge suggests, then the energy difference between the normal state of the neutral *CO* molecule and the initial state of the 0–0 band of the Ångström system would be 86,960 wave-numbers. According to the $h\nu$ relationship, this means the electronic energy associated with the 0–0 transition of the Ångström band system should be 10.73 volts. The observed critical potential for this system is 10.7 volts.

It will be noticed that the neutral *CO* molecule possesses two parallel series of electronic energy levels. This may very possibly mean that the *CO* molecule has two distinct types of spectral terms, corresponding to different types of electronic orbits. Whether or not combination band systems due to transitions between these two sets of levels are possible cannot be said. No such systems have been observed.

Considering along with R. S. Mulliken[2] the analogy between the band spectra just described and the spectra of the corresponding atom, a reasonable explanation can be proposed for the complexity of the third positive system. If each atom retains its own electrons in the K shell but shares those in the other shells, then the *CO* molecule would possess two valence electrons. Its molecular weight is 28. Hence according to the atomic analogy one would expect the spectra of the *CO* molecule to bear some similarity to the spectra of the magnesium atom, the spectral terms of which are singlets

[1] *Nature*, **117**, 230, 1926.

[2] *Physical Review*, **26**, 561, 1925.

and triplets. Pursuing this idea one is led to the possible conclusion that the complex structure of the third positive bands may be due to an overlapping from the several levels of the possible triplet terms. If the separations of the triplet levels are of the same order of magnitude as in the corresponding atom, there would result a superposition of the bands of the several levels causing an apparent complexity in the appearance of the fine structure. The diminution of the triplet separations in the higher levels would result in a simplification in the structure of the bands involving these levels, and this might account for the less complicated nature of the bands of the 3 A system. On the other hand, if we suppose the levels involving the appearance of the fourth positive bands and Angstrom bands to be singlet levels, then one should expect the structure of these bands to be less complex, which is certainly the case.[1]

As stated elsewhere, the observed ionization potential of the molecule was 14.3 volts. The "comet-tail" bands appear at an exciting-voltage of 16.9. Baldet's analysis[2] indicates that 5281 A, corresponding to wave-number 18,930 is the 0–0 band of this system. In terms of equivalent energy, this value represents 2.34 volts which yields 16.64 volts as the exciting-potential of this system. Birge[3] computes 16.7 volts based on 14.2 volts as the ionizing-potential and 4880 A as the 0–0 band. Taking this latter band, 4880 A, as the 0–0 band, with the observed ionization potential, 14.3 volts, the computed critical potential is 16.83 volts.

At 20.0 volts the first negative system appears. Johnson[4] has made a complete analysis of this system and shows without doubt that the band with head at 2190 A represents the 0–0 transition. The emission of this band represents a transition of 5.64 volts, which makes the computed critical potential of this system 19.94 volts—in good agreement with the measured value of 20.0 volts. This work confirms the view that these systems have the same final state, which is the normal state of the ionized molecule.

The observed exciting-potential of the Baldet-Johnson bands

[1] Duffendack and Fox, *Nature*, **118**, 12, 1926.

[2] *Comptes Rendus*, **180**, 272, 1925.

[3] *Nature*, **117**, 230, 1926.

[4] *Proceedings of the Royal Society*, Series A, **108**, 343, 1925.

was 22.9 volts, a value 2.9 volts higher than the exciting-potential of the first negative system. According to the scheme proposed by

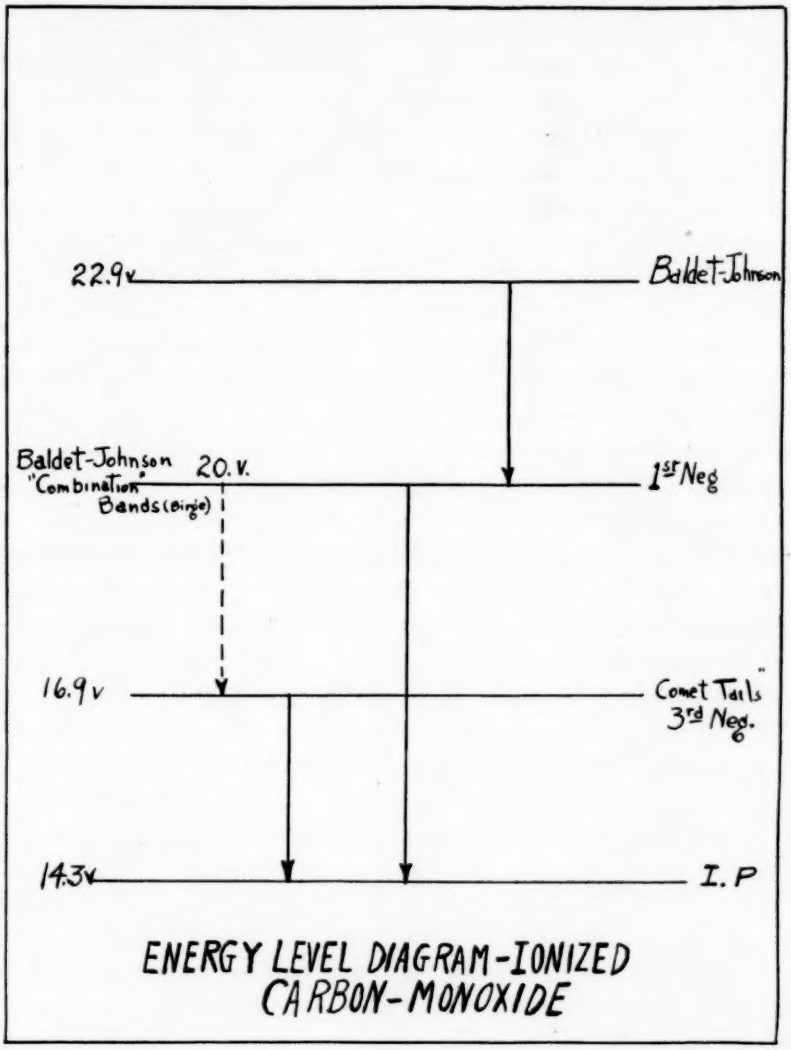


FIG. 4

Birge,[1] this system should be excited at the same critical voltage as the first negative system, based on the observation that the

[1] *Nature*, **117**, 230, 1926.

vibrational differences of the initial states of both these systems seem to follow the same progression. As this system contains so few bands, one cannot be certain of the analysis proposed for it; and hence the band corresponding to the o–o transition is unknown. The energy radiated by the strongest band of the system, 3729.8 A, corresponds to 3.3 volts. As this is not far from the difference in observed exciting-potentials of this system and the first negative, it suggests that the final states of the Baldet-Johnson bands are the initial states of the first negative bands. However, it has not been possible to work out a satisfactory analysis of the system on this basis.

From physical considerations a combination scheme does not seem likely. It will be remembered that both the "comet-tail" and first negative systems were characterized as low-pressure bands, the "comet-tail" system especially so. On the other hand the Baldet-Johnson bands are produced only at high pressures of the order of a millimeter. They completely disappear at pressures at which the first negative and "comet-tail" systems are best developed. This phenomenon appears best explained by the introduction of new levels, since it seems difficult to reconcile the behavior of the Baldet-Johnson bands with two other systems so different. Recently Ann B. Hepburn (November, 1926, meeting of the American Physical Society) reported the excitation potential of the Baldet-Johnson bands as 23.0 volts in agreement with the results of this investigation. Her measurements of the excitation potentials of the Ångström, first negative and "comet-tail" systems, are also in excellent agreement with the values given above.

PART III. DETERMINATION OF THE ORIGIN OF THE "CARBON-OXIDE" BANDS

Throughout the previous discussion it has been tacitly assumed that all the band systems described belonged to carbon monoxide. Although, it is true, the discharge tube was always filled with pure *CO* gas at the beginning of each run, there was considerable dissociation, as was evidenced by an increasing thickness of carbon deposit over the electrodes and walls of the tube. Likewise the necessity of frequent renewal of the filament indicated the presence

of oxygen. The possibility therefore of the presence of CO_2, formed during the passage of the discharge through the tube, did not seem beyond question.

$$2CO \rightarrow CO_2 + C$$
$$2CO \rightarrow O_2 + 2C$$
$$2CO_2 \rightarrow 2CO + O_2$$

I. Langmuir[1] has definitely shown, however, that *CO* is not dissociated by thermal action at the surface of a tungsten filament, even when heated to its melting point. The first two reactions cannot occur except through the influence of the discharge.

To test whether any of the observed spectra were due to CO_2, a discharge tube was connected to a flow apparatus. The filament was, in this case, made of 15 mil platinum wire covered with a heavy coating of calcium, barium, and strontium oxides. This type of filament will give an abundant emission of electrons at a comparatively low temperature and would, therefore, reduce to a minimum the dissociation of CO_2 due to thermal action. Mounted directly in front of this filament was a grid of platinum gauze covering a small window in a sheet of molybdenum which divided the tube into two compartments. The plate of tungsten was mounted about 1 cm from the grid, and the two were then electrically connected. Radiation from the force-free region passed out of the tube through a quartz window.

Carbon dioxide was generated in a vacuum by allowing concentrated sulphuric acid to drop on specially prepared calcium carbonate. The gas thus generated was allowed access to reservoirs which filled to atmospheric pressure. A calibrated stopcock permitted the adjustment of the rate of flow when the pumps were operating so that very uniform pressure conditions could be obtained and maintained indefinitely. Any *CO* that might have been present was removed by freezing out all the CO_2 in the reservoirs with liquid air and opening the reservoirs to the pumping system through a stopcock. As a matter of fact, however, there was no *CO* present, as the gauge read zero pressure both before and after this stopcock was opened.

[1] *Chemical Society Journal*, **37**, 1162, 1925.

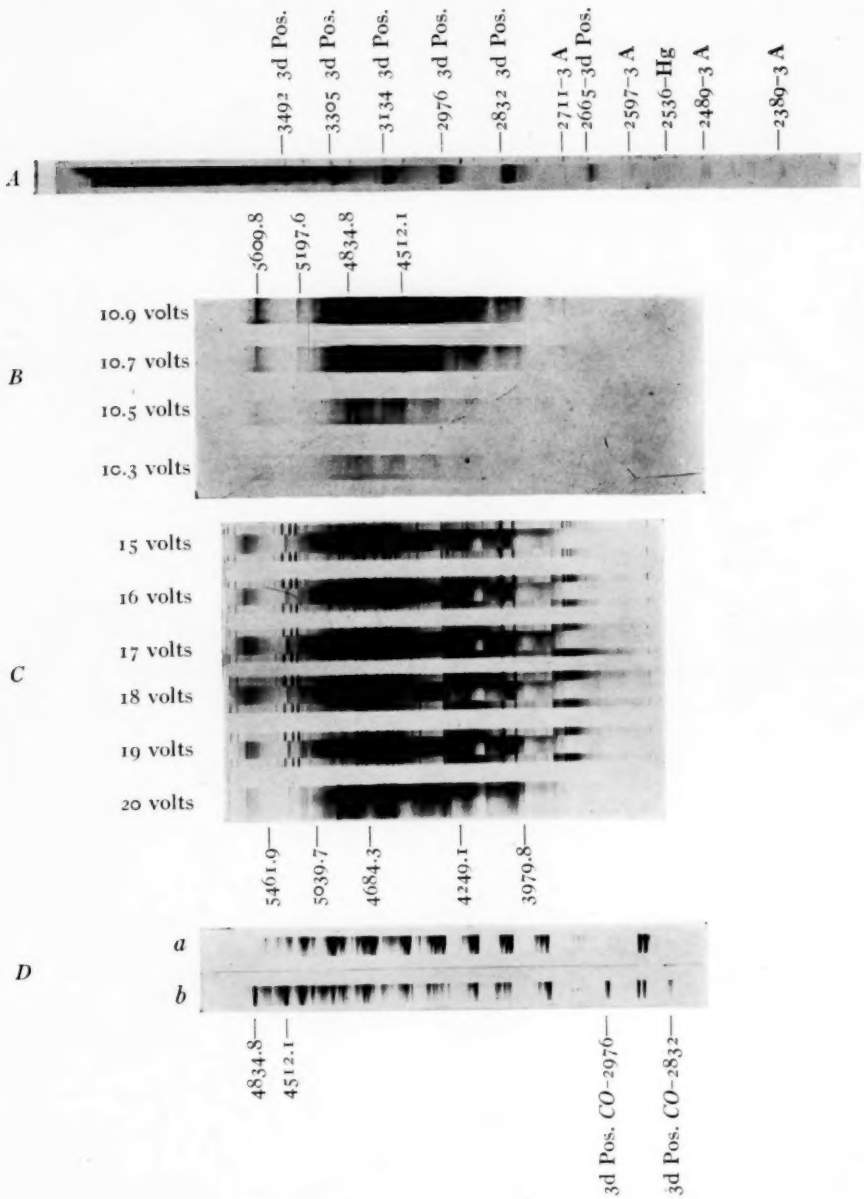
The CO_2 gas entered the discharge tube at the plate end in such a way as to flow against the electron stream, and was then lead out directly above the grid. Thus the radiation examined was that from gas which had not come in contact with the filament, and so was not dissociated by thermal action. After steady pressure conditions had been obtained, the discharge was started and the radiation examined with the spectroscope. Observations were also made with the flow stopped, i.e., in the stagnant gas.

RESULTS

There was a very striking dissimilarity in the color of the discharge in the flowing and stagnant gas. In a previous part of this paper the carbon-monoxide glow was characterized as "sky-blue." The discharge in the CO_2 bore no similarity to this color. It was predominantly purple. The effect of stopping the flow was to change, at once, the color of the discharge from purple to the light blue of carbon monoxide. Likewise in the flowing gas, by increasing the voltage to values of the order of 50 volts, the color of the discharge could be changed from purple over to the blue hue.

Photographs were made with the E2 Hilger spectrograph and with a two-prism glass instrument. The examination of these spectrograms discloses some very interesting information relative to the spectra in the flowing and stagnant gas. With the gas flowing, there was no radiation observed between 2750 A and the limit of the quartz. From 2750 A, however, up through the visible, the spectrum of CO_2 appears filled with a band system of very complicated structure. Groups made of multiple heads, shaded decidedly toward the red, follow along in close sequence. So far as is known, this spectrum is not recorded. In the flowing gas at voltages which should have excited the third positive carbon bands very easily, not a trace of these bands appeared. As soon as the flow was stopped however, not only did the third positive bands appear very strongly but also the Ångström bands and first negative bands, while the spectrum due to the flowing gas appeared suppressed in intensity. At the same time the pressure in the system with the stagnant gas increased, as would be expected, from the conversion of CO_2 into CO and O_2. The foregoing observations lead to the conclusion that the

PLATE VI



A.—Bands of third positive system of *CO* and 3 A bands of *CO*
B.—Showing development of Ångström Bands through Critical Range
C.—Development of "Comet-Tail" Bands through Critical Voltage Range
D.—*a*) CO_2 Flowing Gas
b) CO_2 Stagnant Gas

third positive system of carbon is undoubtedly due to the carbon-monoxide molecule.

In re-examining some of the plates of the *CO* spectrum in the visible, a peculiar striated appearance of the Ångström bands was noted on some exposures. The explanation seems to be that CO_2 was produced in some cases by the action of the discharge, and the striations are due to the superposition on the Ångström bands of the CO_2 spectrum just described. This may mean that several of the originally reported Ångström bands that were not observed during this investigation really do not belong to this system at all but were due to the excitation of CO_2 formed in the Geissler-tube discharge.

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THE MOTIONS OF GIANT M STARS[1]

By GUSTAF STRÖMBERG

ABSTRACT

The *velocities* of 278 giant M stars have been computed from the proper motions, radial velocities, and spectroscopic parallaxes, and projected on three fundamental axes, x, y, and z. The x-axis is the axis of preferential motion, and the y-axis is the axis of asymmetry; the z-axis is nearly perpendicular to the galactic plane. The velocities are referred to the sun as origin.

Formulae for computing the *effect of errors* in absolute magnitude, proper motion, and radial velocity upon the velocity distribution are derived (par. 3). These formulae are applicable to any form of continuous distribution function for the velocities, provided the errors are small.

The distribution of the *apparent velocity components* (i.e., computed without regard to errors) n_o, for intervals of 5 km/sec., is shown in Table I. The *computed* numbers n_c are derived by integration from analytical distribution functions for the true velocities, to the results of which the effect of the errors is added.

The *distribution* of the *x-components* (par. 4) is well represented by a normal error-curve. It has much larger dispersion than the distribution of the y- and z-*components*, but, nevertheless, shows *no trace* of Kapteyn's *two streams*. The true axis of preferential motion for these stars is *perpendicular* to the *axis of asymmetry*. The *distribution* of the *y-components* (par. 5) can be well represented by a function of the type: $A \exp[-a|y-c|-p(y-c)]$. The value for the *asymmetry constant*, $p=+0.0206$ sec./km, is in good agreement with that found for cosmical objects in general, as is also the value of the constant $c=-9.0$ km/sec. No trace of any *discontinuity* can be found in this distribution. The *distribution* of the *z-components* (par. 6) is represented by an error function with a large *positive excess*. Graphical representations of the apparent distribution curves and of the effects of errors are shown in Figs. 1, 2, and 3 by full-drawn curves. For comparison the diagrams also show the observed numbers of velocities within uniform intervals. The dotted curves are representations of the same data based upon distribution functions computed by the *method of moments*. This method gives a poor representation for the y- and z-components.

The *two-dimensional distribution* (par. 7) in the galactic plane can be well represented by a product of the distributions along the x- and y-axes. This representation is shown in Table II, where the upper figures are computed numbers and the lower figures observed numbers. No trace of any discontinuity is indicated; in particular, the high velocities are as well represented as the low velocities. A similar representation is possible for the velocities in the xz- and yz-planes, respectively, which indicates that the velocity components projected on the principal axes are independent of one another.

The velocity components for the "limiting center" are given in par. 8, and the sun's velocity relative to the group of stars studied in par. 9.

Some points in connection with the nature of the general asymmetry in stellar motions are discussed (par. 10); in particular, *the continuous nature of the asymmetry* is emphasized, in contradistinction to another representation of the same phenomenon advocated by Oort. The possible *physical nature* of the asymmetry as suggested by the writer, by Oort, and by Lindblad, is briefly discussed.

1. Different classes of stellar objects show differences both in group motion and velocity dispersion, and it is for this reason important to study the motions of homogeneous groups of objects. In

[1] *Contributions from the Mount Wilson Observatory*, No. 332.

particular, it is of importance to study groups of objects which are not selected on any basis directly correlated with motion. Stars selected on the basis of large proper motion are often included in studies of radial velocities, and, since the dispersion in linear tangential motion and in radial velocity are correlated, the dispersion in the radial velocity is affected by the selection. The effect is even larger and more complicated when three-dimensional motions are studied. In the determination of the asymmetry in stellar motions it is of particular interest to study a homogeneous group of objects having comparatively large velocity dispersion, and the giant M stars seem to be particularly suited to this purpose.

2. In *Mount Wilson Contribution* No. 319,[1] Adams, Joy, and Humason have published absolute magnitudes of practically all known M stars brighter than apparent magnitude 6.0 and north of declination $-30°$. All of these stars are giants, and were selected on the basis of spectral type and apparent brightness alone. The writer has derived the reduction curves for these stars and also the probable errors in the absolute magnitudes.[2] A large number of the radial velocities have been determined at the Lick Observatory and the rest at the Mount Wilson Observatory. The three velocity components of these stars have been computed from the radial velocities, parallaxes, and proper motions, the latter being taken from Boss's *Catalogue*. The result of this study is given in the following pages.

The formulae used for computing space velocities have been published in *Contribution* No. 245.[3] The absolute magnitudes were taken without any correction from *Contribution* No. 319. The proper motions have been corrected for the systematic errors in right ascension as given in Boss's *Catalogue*, page xxviii. To the proper motions in declination, H. Raymond's[4] corrections were applied. One of the principal objects being to study the asymmetry in the velocity distribution, the velocities were projected on three axes, one of which was parallel to the asymmetry axis as determined in *Contribution* No. 293,[5] while the others were in a plane perpendicular

[1] *Astrophysical Journal*, **64**, 225, 1926.

[2] *Mt. Wilson Contr.*, No. 327; *Astrophysical Journal*, **65**, 108, 1927.

[3] *Astrophysical Journal*, **56**, 265, 1922.

[4] *Astronomical Journal*, **36**, 129, 1926.

[5] *Astrophysical Journal*, **61**, 379, 1925.

to this axis with the x-axis in the galactic plane. The right ascension and declination (α, δ), and the galactic longitude and latitude (l, b), of the three axes are:

Axis	α	δ	l	b
x.....	265°.7	−24°.9	331°.5	0°.0
y.....	309.8	+57.1	61.5	+ 9.0
z.....	185.4	+20.0	241.5	+81.0

The y-axis is the axis of asymmetry; the x-axis is nearly parallel to the direction of stream motion; and the z-axis is nearly perpendicular to the galactic plane. The longitudes and latitudes are referred to the standard galaxy, the pole of which has the co-ordinates $\alpha = 190°$, $\delta = +28°$.

3. The computed velocities are affected by errors in the absolute magnitudes, proper motions, and radial velocities. The effect of errors in the absolute magnitudes has been studied by J. H. Oort[1] on the assumption that the true velocity distribution is a normal error function. In the present case we need expressions for the effect of all three classes of errors, without making any particular assumption as to the form of the true frequency function of the velocities; and we also need expressions for the effect of errors integrated over arbitrary intervals.

We will assume that the errors Δ in the adopted absolute magnitudes M are distributed according to a normal error-curve around a zero value. The same will be assumed for the errors ϵ and ν in the proper-motion components μ and the radial velocities V. Denoting true values by subscripts, we have

$$M_1 = M + \Delta\ ; \qquad \mu_1 = \mu + \epsilon\ ; \qquad V_1 = V + \nu\ . \tag{1}$$

For the parallaxes p we have

$$\left.\begin{aligned} p_1 &= p\,10^{0.2\Delta} = p e^{q\Delta} \\ q &= \frac{0.2}{\mathrm{Mod}} = 0.4605\ . \end{aligned}\right\} \tag{2}$$

[1] *Groningen Publications*, No. 40, p. 17, 1926.

Projecting the proper motions upon an arbitrary axis x perpendicular to the line of sight, we have for the linear motion along this axis the expression

$$x_1=\frac{k\mu_1}{p_1}=\frac{k(\mu+\epsilon)}{p}\,e^{-q\Delta}=(x+\epsilon')e^{-q\Delta}\,, \tag{3}$$

where $k=4.738$ km$\times$year/sec., and ϵ' is the error in the proper-motion component converted into km/sec. The distribution function of the errors Δ and ϵ are supposed to have the moduli h and h_1. Let $F_1(x_1)$ be the frequency function of the true velocities projected on a certain axis. The frequency function $F(x)$ of the apparent tangential velocity components (i.e., computed without regard to the effect of errors) is then found from the equation

$$F(x)dx=\frac{h\,h_1}{\pi}\iint_{\Delta,\,\epsilon=-\infty}^{+\infty}F_1(x_1)e^{-h^2\Delta^2-h_1^2\epsilon^2}\,d\Delta\,d\epsilon\,dx_1\,, \tag{4}$$

where x_1 has the value given in equation (3). We have

$$d\Delta\,d\epsilon\,dx_1=e^{-q\Delta}\,d\Delta\,d\epsilon\,dx\,.$$

Putting

$$h\Delta=z$$

and

$$\frac{q}{2h}=\beta\,,$$

we find

$$F(x)dx=\frac{dx\,h_1}{\pi}\,e^{\beta^2}\iint_{-\infty}^{+\infty}e^{-(z+\beta)^2-h_1^2\epsilon^2}\,F_1[(x+\epsilon')e^{-2\beta z}]dz\,d\epsilon\,. \tag{5}$$

As will appear later, the quantities β and ϵ', the latter being expressed in units equal to the dispersion in x, are small quantities, and we can develop the function F_1 under the integration sign in a power series, limiting ourselves to quantities of the second order in β and ϵ'. We derive thus

$$\left.\begin{aligned}F_1[(x+\epsilon')e^{-2\beta z}]=F_1(x)+\epsilon'\frac{dF_1}{dx_1}-2\beta zx\frac{dF_1}{dx_1}+\frac{\epsilon'^2}{2}\frac{d^2F_1}{dx_1^2}-2\beta\epsilon' z\frac{dF_1}{dx_1}\\ +2\beta^2z^2\left[x^2\frac{d^2F_1}{dx_1^2}+x\frac{dF_1}{dx_1}\right].\end{aligned}\right\} \tag{6}$$

From equation (5) we now find

$$F(x)=F_1(x)+\beta^2\left[F_1(x)+3x\frac{dF_1}{dx_1}+x^2\frac{d^2F_1}{dx_1^2}\right]+\frac{k^2\overline{\epsilon^2}}{2\overline{p_1^2}}\frac{d^2F_1}{dx_1^2}. \qquad (7)$$

The quantity β is connected with the dispersion r in the absolute magnitudes by the equation

$$\beta=\frac{qr}{\sqrt{2}}.$$

For the particular case that $F_1(x)$ is a normal error function and $\overline{\epsilon^2}=0$, we may derive Oort's expression from equation (7).

The effect of the errors ν in the radial velocities can be found in exactly the same way as in the case of ϵ'. If we assume that the dispersion in the errors is small as compared with that in the velocities, we find the effect of these errors to be

$$E'(x)dx=\frac{\overline{\nu^2}}{2}\frac{d^2F_1}{dx_1^2}. \qquad (8)$$

The errors in the velocities projected on a fixed axis x are the combined effect of errors in tangential and radial motions. For a uniform distribution of the stars over the sky, or over one hemisphere, we can find the total effect by multiplying equations (7) and (8) by 2:3 and 1:3, respectively. The total effect of the errors in absolute magnitude, proper motion, and radial velocity is thus

$$\left.\begin{aligned} E(x)dx&=\left[a\left\{F_1(x)+3x\frac{dF_1}{dx_1}+x^2\frac{d^2F_1}{dx_1^2}\right\}+\frac{T^2}{2}\frac{d^2F_1}{dx_1^2}\right]dx \\ a&=\frac{q^2r^2}{3}\;;\quad T^2=\frac{2}{3}\frac{k^2\overline{\epsilon^2}}{\overline{p_1^2}}+\frac{\overline{\nu^2}}{3}. \end{aligned}\right\} \qquad (9)$$

In this equation we may replace $\overline{p_1^2}$ by the mean of the squares of the assumed parallaxes. The velocities x must here be referred to the sun as origin.

The function $F(x)dx=F_1(x)dx+E(x)dx$ can be integrated over finite intervals and compared with the observed distribution. Such expressions will later be derived for three types of distribution functions. In the case of a normal error-curve the last term in the expression for $E(x)$ in equation (9) may be dropped and replaced by

an increase in the square of the dispersion by an amount equal to T^2. Although this rule is also applicable to more complex forms of distribution functions, the dispersion will not in these cases be used as one of the constants of the distribution law.

The value used for the mean error r in the absolute magnitudes is ± 0.75, based upon a probable error ± 0.5 derived in *Contribution* No. 327. This may be regarded as a mean value for the giants and supergiants. We have thus $a = 0.0398$. The mean error in the proper-motion components has been assumed to be $\pm 0''.005$, and that in the radial velocities ± 3.0 km/sec. The value found for the square root of $\bar{p}^2$ is $0''.0071$. We have thus $T = 3.22$ km/sec., which is small compared to the dispersion in velocity; the expansion is consequently valid.

4. A total number of 278 stars was studied. No difference could be found in the motions of the giants and supergiants, and they were consequently studied together.

The distribution of the velocities projected on the x-axis showed, when plotted, a great resemblance to a normal error-curve. The apparent distribution was first represented with the aid of the method of moments. This method has been developed and extensively used by Charlier and his co-workers at the Lund Observatory. Analytically it is very beautiful, but has the disadvantage of giving stars of high velocity much higher weight than those of the smaller velocities. Although the distribution function represents the moments exactly, it does not follow that it is the best representation for the actual distribution which can be obtained with the same number of parameters. The formulae used in this method are given below.

The distribution is represented by the function

$$F(\xi) = \phi + \beta_3 \frac{d^3\phi}{d\xi^3} + \beta_4 \frac{d^4\phi}{d\xi^4} \qquad \phi = \frac{1}{\sqrt{2\pi}} e^{-\xi^2/2}, \qquad \xi = \frac{x-c}{\sigma} \tag{10}$$

The dispersion is here represented by σ and the mean by c. It is customary to limit one's self to moments of the fourth order. Denoting the moments of order n by v_n, the moments are defined by the expression

$$v_n = \sigma^n \int_{-\infty}^{+\infty} \xi^n F(\xi) d\xi ,$$

which is the analytical expression for the mean of the nth power of the differences $x-c$. In the numerical computation of the moments, we use the expression

$$v_n=\overline{(x-c)^n}\,.$$

It is obvious that the moments of high order are extremely uncertain, being greatly affected by a few large values. The coefficients β_3 and β_4 represent the skewness and the excess of the distribution, respectively, and are connected with the moments of third and fourth order by the expressions

$$\beta_3=-\frac{v_3}{6\sigma^3}\;;\qquad \beta_4=\frac{1}{24}\left(\frac{v_4}{\sigma^4}-3\right). \tag{11}$$

The skewness S and the excess E are defined by the equations

$$S=3\beta_3\;;\qquad E=3\beta_4\,. \tag{12}$$

Tables for $\phi(\xi)$, $d^3\phi/d\xi^3$, and $d^4\phi/d\xi^4$ have been published by C. V. L. Charlier,[1] which greatly facilitate the computation of the distribution from the constants.

For the distribution of the apparent velocities projected on the x-axis, I find from the moments

$$\left.\begin{aligned} c&=-\ 9.19\ \text{km/sec.} & S&=+0.028\\ \sigma&=\ 31.34\ \text{km/sec.} & E&=+0.202 \end{aligned}\right\} \tag{13}$$

Figure 1 shows the frequency curve based on these constants as a dotted curve, together with the observed numbers of velocities in intervals of 5 km/sec. The representation by the use of moments is in this case very good.

Since the positive excess thus found might possibly be entirely due to the errors in the velocities, an attempt was made to represent the true distribution by a normal error curve. For this the constants were determined from the distribution itself according to a method similar to that used in *Contribution* No. 245,[2] which involves the determination of approximate values of the constants, subsequently

[1] *Vorlesungen über die Grundzüge der Mathematischen Statistik.* Lund: Verlag Scientia, 1920.

[2] *Astrophysical Journal*, **56**, 265, 1922.

corrected by means of the differences between computed and observed numbers of velocities between different limits. The intervals, ten in number, were so chosen that they included approximately equal numbers of velocities, the equations of condition then being given equal weights.

The assumed frequency function for the apparent velocities is

$$F(x)dx=\frac{Nh}{\sqrt{\pi}}e^{-h^2(x-c)^2}dx+E(x)dx\ , \tag{14}$$

where N is the number of stars and $E(x)$ has the value given in equation (9).

Integrating between the limits x_1 and x_2, we find

$$\left.\begin{aligned} &n_c=\int_{x_1}^{x_2}F(x)dx=\frac{N\Theta}{2}+\frac{N}{\sqrt{\pi}}[b_0\psi_0+b_1\psi_1+b_2\psi_2+b_3\psi_3]\\ &\Theta=\frac{2}{\sqrt{\pi}}\int_{t_1}^{t_2}e^{-t^2}dt\ ;\quad t_1=h(x_1-c)\ ;\quad t_2=h(x_2-c)\\ &\psi_n=t_1^ne^{-t_1^2}-t_2^ne^{-t_2^2}\\ &b_0=-a\,hc\ ;\qquad\qquad b_2=4a\,hc\\ &b_1=a(-1+2h^2c^2)+T^2h^2\ ;\qquad b_3=2a \end{aligned}\right\} \tag{15}$$

The expressions for the derivatives are

$$\frac{dn_c}{dh}=-\frac{N}{h\sqrt{\pi}}\psi_1\ ;\qquad \frac{dn_c}{dc}=\frac{Nh}{\sqrt{\pi}}\psi_0\ , \tag{16}$$

and the equations of condition for the correction of the constants are

$$\Delta h\frac{dn_c}{dh}+\Delta c\frac{dn_c}{dc}=n_0-n_c\ ,$$

where n_0 and n_c are the observed and computed numbers of velocities, respectively, lying between the limits x_1 and x_2.

The following values and probable errors for the mean c and the dispersion σ for the true velocity distribution were thus derived:

$$\left.\begin{aligned} c&=-8.78\pm0.68\ \text{km/sec.}\\ \sigma=\frac{1}{\sqrt{2h}}&=\ 27.58\pm0.55\ \text{km/sec.} \end{aligned}\right\} \tag{17}$$

TABLE I

Velocity km/sec.	x–Axis					y–Axis					z–Axis				
	$\int F_1(x)dx$	$\int E(x)dx$	n_c	n_o	O–C	$\int F_1(y)dy$	$\int E(y)dy$	n_c	n_o	O–C	$\int F_1(z)dz$	$\int E(z)dz$	n_c	n_o	O–C
$-\infty$															
−110	0.03	+0.29	0.32	1	+0.7	1.16	+1.13	2.29	0	−2.3					
−105	0.04	+0.19	0.23	0	−0.2	0.33	+0.18	0.51	0	−0.5				0	0.0
−100	0.06	+0.27	0.33	1	+0.7	0.42	+0.20	0.62	1.5	+0.9					
− 95	0.13	+0.38	0.51	0	−0.5	0.56	+0.23	0.79	1.5	+0.7				1	+1.0
− 90	0.19	+0.52	0.71	1	+0.3	0.68	+0.23	0.91	0	−0.9		0.0	0.00	0	0.0
− 85	0.35	+0.63	0.98	1	0.0	0.88	+0.25	1.13	0	−1.1	0.00			0	0.0
− 80	0.57	+0.74	1.31	2	+0.7	1.13	+0.25	1.38	1.5	+0.1				0.5	+0.5
− 75	0.92	+0.79	1.71	1	−0.7	1.46	+0.27	1.73	1	−0.7				0.5	+0.5
− 70	1.39	+0.74	2.13	3	+0.9	1.86	+0.25	2.11	1.5	−0.6				0	0.0
− 65	2.10	+0.59	2.69	3	+0.3	2.38	+0.24	2.62	3	+0.4		+0.01	0.01	1	+1.0
− 60	3.03	+0.31	3.34	2	−1.3	3.06	+0.19	3.25	4	+0.8		+0.04	0.04	0	0.0
− 55	4.24	−0.08	4.16	3.5	−0.7	3.91	+0.15	4.06	4.5	+0.4	0.02	+0.19	0.21	1	+0.8
− 50	5.73	−0.53	5.20	4.5	−0.7	5.00	+0.04	5.04	3.5	−1.5	0.07	+0.59	0.66	0.5	−0.2
− 45	7.52	−0.95	6.57	7	+0.4	6.42	−0.04	6.38	7	+0.6	0.33	+1.26	1.59	2.5	+0.9
− 40	9.54	−1.28	8.26	8	−0.3	8.21	−0.22	7.99	8.5	+0.5	1.21	+2.32	3.53	1.5	−2.0
− 35	11.68	−1.42	10.26	11	+0.7	10.52	−0.32	10.20	10	−0.2	2.49	+0.77	3.26	3	−0.3
− 30	13.89	−1.34	12.55	14	+1.4	13.47	−0.49	12.98	16.5	+3.5	5.56	−0.39	5.17	4.5	−0.7
− 25	15.96	−1.05	14.91	14.5	−0.4	17.25	−0.62	16.63	17	+0.4	9.51	−1.70	7.81	5.5	−2.3
− 20	17.75	−0.60	17.15	15	−2.2	22.10	−0.68	21.42	25.5	+4.1	14.04	−1.29	12.75	14	+1.2
− 15	19.10	−0.09	19.01	19	0.0	28.31	−0.58	27.73	28.5	+0.8	19.40	−0.52	18.88	18	−0.9
− 10	19.90	+0.34	20.24	17.5	−2.7	36.27	−0.21	36.06	30	−6.1	26.41	−0.80	25.61	21	−4.6
− 5	20.06	+0.62	20.68	22	+1.3	37.25	(+2.52)	(39.77)	40	(+0.2)	34.00	−0.84	33.16	42.5	+9.3
0	19.57	+0.69	20.26	23	+2.7	24.05	(+5.39)	(29.44)	26.5	(−2.9)	38.28	−0.15	38.13	38	−0.1

TABLE I—*Continued*

Velocity km/sec.	x−Axis					y−Axis					z−Axis				
	$\int F_1(x)dx$	$\int E(x)dx$	n_c	n_o	O−C	$\int F_1(y)dy$	$\int E(y)dy$	n_c	n_o	O−C	$\int F_1(z)dz$	$\int E(z)dz$	n_c	n_o	O−C
0	18.47	+0.54	19.01	18.5	−0.5	15.28	+0.91	16.19	14.5	−1.7	36.17	−0.03	36.14	29.5	−6.6
+ 5	16.87	+0.24	17.11	21	+3.9	9.71	+0.21	9.92	15.5	+5.6	29.23	−0.30	28.93	29.5	+0.6
+ 10	14.91	−0.12	14.79	13	−1.8	6.18	0.00	6.18	4.5	−1.7	21.73	−0.27	21.46	23.5	+2.0
+ 15	12.77	−0.44	12.33	14	+1.7	3.91	−0.02	3.89	4	+0.1	15.82	−0.75	15.07	17.5	+2.4
+ 20	10.58	−0.64	9.94	8.5	−1.4	2.52	+0.01	2.53	1.5	−1.0	11.09	−1.50	9.59	9	−0.6
+ 25	8.47	−0.70	7.77	5	−2.8	1.56	+0.05	1.61	3.5	+1.9	6.90	−0.99	5.91	4.5	−1.4
+ 30	6.57	−0.61	5.96	3.5	−2.5	1.01	+0.07	1.08	1	−0.1	3.57	+0.53	4.10	6.5	+2.4
+ 35	4.93	−0.42	4.51	6.5	+2.0	0.64	+0.09	0.73	0	−0.7	1.50	+1.44	2.94	0	−2.9
+ 40	3.57	−0.19	3.38	4	+0.6	0.40	+0.08	0.48	0	−0.5	0.49	+1.28	1.77	1	−0.8
+ 45	2.52	+0.04	2.56	2	−0.6	0.26	+0.08	0.34	0	−0.3	0.14	+0.71	0.85	1	+0.2
+ 50	1.71	+0.22	1.93	1.5	−0.4	0.17	+0.08	0.25	1.5	+1.2	0.03	+0.28	0.31	0	−0.3
+ 55	1.14	+0.34	1.48	2	+0.5	0.10	+0.05	0.15	0.5	+0.4	0.00	+0.06	0.06	1	+0.9
+ 60	0.71	+0.38	1.09	1.5	+0.4	0.07	+0.05	0.12	0	−0.1		+0.02	0.02	0	0.0
+ 65	0.44	+0.37	0.81	0	−0.8	0.03	+0.02	0.05	0	0.0					
+ 70	0.26	+0.34	0.60	1	+0.4	0.03	+0.03	0.06	0	−0.1					
+ 75	0.15	+0.27	0.42	0	−0.4	0.02	+0.02	0.04	0	0.0					
+ 80	0.08	+0.20	0.28	1	+0.7	0.01	+0.02	0.03	0	0.0	0.00	0.00	0.00	0	0.0
+ 85	0.06	+0.16	0.22	0	−0.2										
+ 90	0.01	+0.10	0.11	0	−0.1	0.02	+0.04	0.06	0	−0.1					
+ 95	0.01	+0.06	0.07	0	−0.1										
+100	0.01	+0.09	0.10	1	+0.9										
+ ∞															

The apparent distribution, that is, the true distribution plus the effect of the errors, is shown as a continuous curve in Figure 1. The effect of the errors is shown as another continuous curve near the axis of the abscissae. The numerical values are given in Table I.

The representation of the observed data is perfectly satisfactory, and no excess is now found for the true distribution. In fact, the excess for the true distribution was actually computed by a method explained later in connection with the z-components, and, as measured by the quantity γ_4, was found to be $+0.002 \pm 0.044$, which means that no excess is present after the distribution has been corrected for the effect of errors of observation.

An attempt was also made to determine the mean error in the absolute magnitudes by assuming that the true distribution is a normal error function. The result is $r = \pm 0.75 \pm 0.28$, in exact agreement with the adopted value, but with a fairly large uncertainty.

The x-axis is nearly in the direction of preferential motion as ordinarily given. Thus the direction of the latter as found by A. S. Eddington[1] from the proper motions of the Boss stars analyzed on the basis of the two-stream theory is toward galactic longitude 347°, whereas the x-axis has a longitude of 332°, both axes having zero galactic latitudes. From Eddington's data, I have computed the position of the centers of the two streams in the frequency curve and found that the first stream has its center at the point -15.7 km/sec. and the second stream at $+23.5$ km/sec. Allowing for the slight inclination between the two axes, we should expect the center of the first stream at the point -15.2 and of the second stream at $+22.7$. These points are marked with roman numerals in Figure 1. No trace of any division into two streams is noticeable, although the "stream" motion is very pronounced among these stars, as shown by a comparison of the dispersions along the three axes. For the giant M stars the evidence is thus all in favor of the ellipsoidal hypothesis.

The direction of the axis of preferential motion for the "central group" among the A stars, for the giant stars of spectral classes K and M, and also for the dwarf stars of the latter types, has been

[1] *Stellar Movements*, pp. 101, 102. Macmillan, 1914.

determined by the writer[1] to be toward longitude 330°, differing appreciably from the direction found for other spectral classes, which is about 357°. The reason for this difference is probably that we have eliminated from the first-mentioned objects the effect of moving groups, in particular the generalized Taurus group and the Ursa Major group, and any existing traces of Kapteyn's two streams. A direct computation from the present data using the method devised in *Contribution* No. 245 gives $332^{\circ}.0 \pm 2^{\circ}.1$ for the galactic longitude of the axis of preferential motion, in exact agreement with

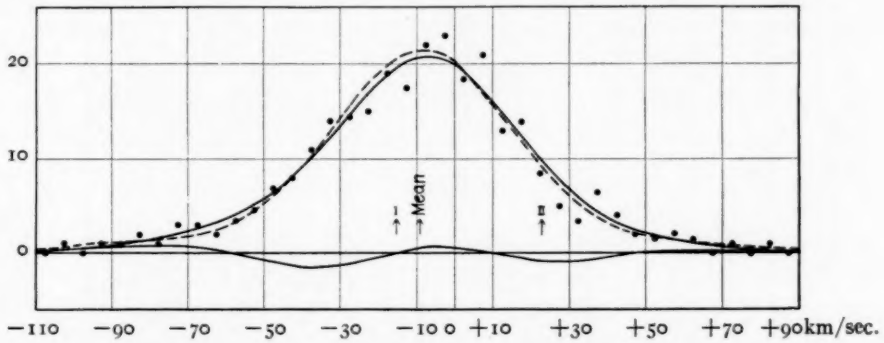


FIG. 1.—Distribution of velocities projected on the axis of preferential motion. The dots represent observed numbers of velocities between limits of 5 km/sec. The dotted curve is a representation determined by the method of moments; the full drawn curve is based upon an analytical distribution function to which the effect of errors has been added. The curve near the axis of abscissae represents the effect of errors in the absolute magnitudes, proper motions, and radial velocities.

the direction used. For the stars studied the axis of preferential motion is thus perpendicular to the axis of asymmetry, at least within the uncertainty of the directions involved.

5. The y-axis is the axis of asymmetry as determined in *Contribution* No. 293. The observed frequency function was first represented by the method of moments, values for the mean, dispersion, skewness, and excess being found as follows:

$$\left.\begin{aligned} c &= -17.43 \text{ km/sec.} & S &= +0.356 \\ \sigma &= 22.59 \text{ km/sec.} & E &= +0.369 \end{aligned}\right\} \qquad (18)$$

[1] *Mt. Wilson Contr.*, Nos. 245, 257; *Astrophysical Journal*, **56**, 265, 1922; **57**, 77, 1923.

The representation of the distribution with the aid of these constants is shown as a dotted curve in Figure 2. The representation is very poor because the actual distribution differs greatly from a normal error-curve. A slightly better representation could perhaps be obtained by omitting a few of the higher velocities; but such omissions are of a rather arbitrary nature, and one of the specific purposes of this study was to find if it were possible to represent the whole distribution by a simple function.

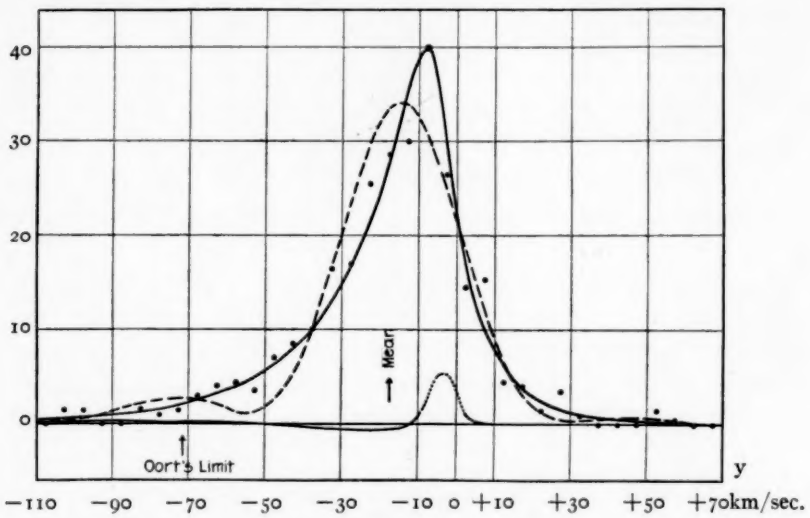


FIG. 2.—Same as Fig. 1, but the velocities are here projected on the axis of asymmetry.

Contributions Nos. 275 and 293 give evidence that the aggregate of practically all cosmical objects has a velocity distribution which is symmetrical for the x- and z-components and asymmetrical for the y-components. The velocity distribution for the y-axis was represented by a function of the form

$$F(y)dy = e^{-p(y-c)}\, S[(y-c)^2]dy\,, \qquad (19)$$

where S indicates a distribution function symmetrical around the value $y=c$, and p is a quantity which produces the asymmetry and may be referred to as the "asymmetry constant"; in reality it is a

vector directed toward the negative y-axis. In the papers cited, S was represented by a sum of concentric normal error-curves, which was a natural representation on account of the heterogeneous mixture of objects used. If S is a normal error-curve, the effect of the asymmetry constant is to shift the center of this error-curve in the direction of the asymmetry vector by an amount which is a quadratic function of the dispersion.

An attempt was made to represent the true distribution by a function of the form given in equation (19). The difficulty was to find an analytical form for the function S. After several trials with error-curves having different kinds of excess, a very simple form was found which represents the true distribution remarkably well.

The form used was

$$S \sim e^{-a|y-c|}, \tag{20}$$

where the absolute value of $(y-c)$ is used in the exponent. This function has a singularity at the point $y=c$, which will be discussed later.

The observed distribution was thus represented by the function

$$F(y)dy = \frac{N(a^2-p^2)}{2a} e^{-a|y-c|-p(y-c)}\,dy + E(y)dy, \tag{21}$$

where $E(y)dy$, as before, is the effect of errors.

Integrating the function $F(y)dy$ between the limits $-\infty$ and y for $y \leq c$, and between y and $+\infty$ for $y \geq c$, we obtain

$$\left.\begin{aligned} n_c' &= \int_{-\infty}^{y} F(y)dy = \frac{N(a+p)}{2a} e^{(a-p)(y-c)}\left[1+a\{(a-p)y+(a-p)^2y^2\}\right. \\ &\qquad \left. +\frac{T^2}{2}(a-p)^2\right], \\ n_c'' &= \int_{y}^{+\infty} F(y)dy = \frac{N(a-p)}{2a} e^{-(a+p)(y-c)}\left[1-a\{(a+p)y\right. \\ &\qquad \left. -(a+p)^2y^2\}+\frac{T^2}{2}(a+p)^2\right]. \end{aligned}\right\} \tag{22}$$

The expressions for the derivatives are

$$\left.\begin{aligned}
&\left.\begin{aligned}
\frac{dn'_c}{da} &= \frac{N}{2a} e^{(a-p)(y-c)}\left[-\frac{p}{a}+(a+p)(y-c)\right] \\
\frac{dn'_c}{dp} &= \frac{N}{2a} e^{(a-p)(y-c)}\left[1-(a+p)(y-c)\right] \\
\frac{dn'_c}{dc} &= -\frac{N}{2a}(a^2-p^2)e^{(a-p)(y-c)}
\end{aligned}\right\} (y \leq c) \\
&\left.\begin{aligned}
\frac{dn''_c}{da} &= \frac{N}{2a} e^{-(a+p)(y-c)}\left[\frac{p}{a}-(a-p)(y-c)\right] \\
\frac{dn''_c}{dp} &= \frac{N}{2a} e^{-(a+p)(y-c)}\left[-1-(a-p)(y-c)\right] \\
\frac{dn''_c}{dc} &= \frac{N}{2a}(a^2-p^2)e^{-(a+p)(y-c)} .
\end{aligned}\right\} (y \geq c)
\end{aligned}\right\} \qquad (23)$$

I have employed the method used in the case of the x-co-ordinates and determined corrections to provisional values of the constants a, p, and c from the differences between observed and computed numbers of velocities in different intervals. These intervals, ten in number, were so chosen that they include approximately equal numbers of velocities.

The expression for $E(y)dy$ is derived on the assumption that the first and second derivatives of the frequency function are continuous. In the present case these derivatives are discontinuous for the value $y=c$. Consequently, we must imagine some kind of continuous transition in these derivatives in the neighborhood of this point. The computation was carried out to the singularity at $y=c$, but the computed value of $\int E(y)dy$ in the region around this point was regarded as uncertain, and the corresponding equation of condition was omitted in the least-squares solution.

The following constants were found:

$$\left.\begin{aligned}
a &= +0.0701 \pm 0.0041 \text{ sec./km} \\
p &= +0.0206 \pm 0.0041 \text{ sec./km} \\
c &= -9.02 \quad \pm 1.03 \quad \text{km/sec.}
\end{aligned}\right\} \qquad (24)$$

The apparent distribution function based upon these constants and integrated over intervals of 5 km/sec. is shown graphically as a full-drawn curve in Figure 2, and the numerical data are given in Table I. The effect of the errors is also shown in the diagram, but the computed effect near the singularity is not exact. The above constants are, however, independent of the errors in this region of the distribution curve.

The representation by means of the foregoing constants is seen to fit the observed data remarkably well throughout the range. The values of p and c are in good agreement with those found in *Contribution* No. 293 for the asymmetry constant and the "limiting center" from cosmical objects in general. The values previously derived are

$$\left.\begin{aligned} p &= +0.0192 \pm 0.0020 \text{ sec./km} \\ c &= -10.0 \quad \pm 1.0 \quad \text{km/sec.} \end{aligned}\right\} \quad (25)$$

In fact, we might have adopted these values and obtained a representation practically identical with the one given, by the use of only one constant (a), the others (p, c, a, and T) being derived from other objects or by other means.

6. The velocity distribution along the z-axis is shown in Table I and Figure 3. This axis is nearly perpendicular to the galactic plane. Using again the method of moments, we find the following constants for the apparent distribution

$$\left.\begin{aligned} c &= -\ 2.33 \text{ km/sec.}\ ; \quad S = +0.419 \\ \sigma &= \ 18.64 \text{ km/sec.}\ ; \quad E = +0.445 \end{aligned}\right\} \quad (26)$$

The dispersion is slightly smaller than that found for the y-axis, but the skewness and excess are very large. The representation by these constants is shown by a dotted curve in Figure 3 and is rather poor. When the excess or the skewness becomes large, pronounced inflections occur and the frequency function may even become negative. To avoid these inflections, which do not appear in the observed distribution, a function of the following form was used:

$$F_1(z)dz = \frac{Nh}{\sqrt{\pi\gamma}} e^{-h^2(z-c)^2} [1 + \gamma_4 h^4 (z-c)^4] dz;\ \gamma = 1 + \frac{3}{4}\gamma_4 . \quad (27)$$

This function gradually becomes zero and is always positive when γ_4 is positive. It must be emphasized that h is not here a measure of the actual dispersion.

The apparent distribution is thus

$$F(z)dz = F_1(z)dz + E(z)dz\ .$$

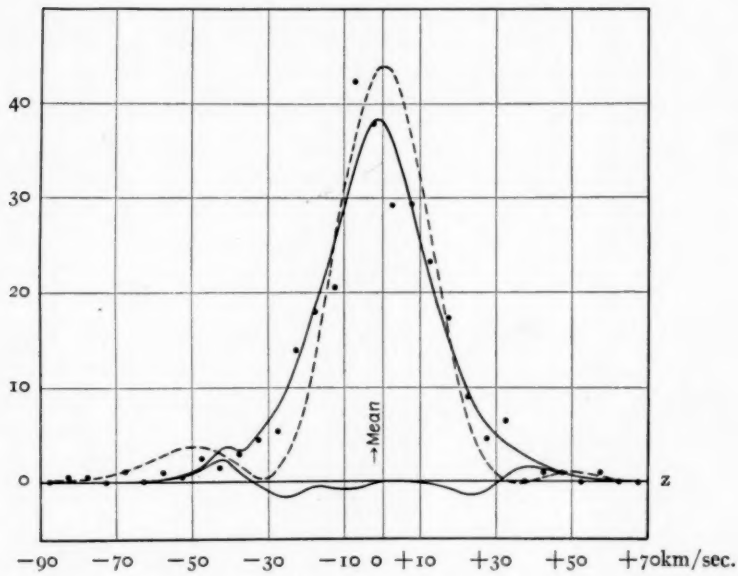


FIG. 3.—Same as Fig. 1, the velocities being projected on an axis nearly perpendicular to the galactic plane.

Integrating this function between the limits z_1 and z_2 we find

$$\left.\begin{aligned}
&n_c = \int_{z_1}^{z_2} F(z)dz = \frac{N\Theta}{2} + \frac{N\gamma_4}{4\gamma\sqrt{\pi}}(3\psi_1 + 2\psi_3) + \frac{N}{\gamma\sqrt{\pi}}\sum_{n=0}^{n=7} b_n\psi_n \\
&b_0 = -ahc \qquad\qquad b_4 = -9ahc\gamma_y \\
&b_1 = a(-1+2h^2c^2) + T^2h^2 \qquad b_5 = a\gamma_y(-5+2h^2c^2) + T^2h^2\gamma_y \\
&b_2 = 4ahc \qquad\qquad b_6 = 4ahc\gamma_4 \\
&b_3 = a(2-4h^2c^2\gamma_4) - 2T^2h^2\gamma_4 \qquad b_7 = 2a\gamma_4
\end{aligned}\right\} \quad (28)$$

The functions Θ and ψ_n are defined by equations (15).

The expressions for the derivatives are

$$\left.\begin{aligned}\frac{dn_c}{dh} &= -\frac{N}{h\sqrt{\pi\gamma}}(\psi_1+\gamma_4\psi_5)\\ \frac{dn_c}{dc} &= \frac{Nh}{\sqrt{\pi\gamma}}(\psi_0+\gamma_4\psi_4)\\ \frac{dn_c}{d\gamma_4} &= \frac{N}{2\sqrt{\pi\gamma^2}}\left(\frac{3}{2}\psi_1+\psi_3\right)\end{aligned}\right\} \tag{29}$$

The following values for the constants were found:

$$\left.\begin{aligned} c &= -\ 1.61\ \pm 0.61 \text{ km/sec.}\\ \sigma' = \frac{1}{\sqrt{2h}} &= \ 11.54\ \pm 1.36 \text{ km/sec.}\\ \gamma_4 &= +\ 0.322 \pm 0.191\ .\end{aligned}\right\} \tag{30}$$

The true dispersion σ can be found from the equation

$$\sigma^2 = \frac{\sigma'^2}{\gamma}\left(1+\frac{15}{4}\gamma_4\right),$$

and we find thus

$$\sigma = 15.39 \pm \text{km/sec.}$$

The representation with these constants is shown in Figure 3 by a full-drawn curve, and may be regarded as fairly satisfactory. The effect of the errors is shown as before by a full-drawn curve near the lines of abscissae. The hump in the distribution at $z = -40$ km/sec. is due to the errors in the absolute magnitudes, as may be seen from the curve representing the effect of the errors.

The true distribution function is slightly asymmetric, and an attempt was made to represent the asymmetry by a factor $e^{-p(z-c)}$, as in the case of the y-components. The slight asymmetry which exists is, however, not of a type that can be represented in this way, and is probably not a general phenomenon.

7. Even if the velocity distributions for the three principal axes are known, it does not necessarily follow that the three-dimensional distribution can be represented by a product of the three functions.

It is only when the velocity components are independent that the simple product may be used. In case, for instance, of a mixture of two classes of objects having different velocity dispersions but the same group motion, there would be a positive correlation between the velocity components, numerically small and large velocity components on one axis being statistically associated with small and large components, respectively, on another axis, a condition[1] which may be expressed by inequalities of the type $\overline{x^2y^2} > \overline{x^2}\,\overline{y^2}$.

In the present case the data are hardly sufficient for a three-dimensional subdivision, and we will limit ourselves to a study of the motions in the xy-plane. If we multiply the apparent distribution functions for the x- and y-axes and divide the result by the total number of objects N, we may test the possibility of representing the velocity distribution in the galactic plane by this product. Table II gives the results of such a study. The elements of area are necessarily increased as the density decreases, in order to obtain enough stars in each element. The upper figure in each rectangle is the computed number of velocity vectors terminating within the corresponding element of the xy-plane, while the lower figure is the observed number. On account of the use of even kilometers in the individual velocity components, the observed numbers often include fractions of a star. In view of the comparatively small number of stars in the different elements, the agreement between computed and observed values is all that can be expected.

It was of particular interest to ascertain whether the high velocities show any evidence of discontinuity. A little below the x-axis there is an excess of four or five stars with velocities beyond -80 and $+70$ km/sec., but the opposite occurs for the $y < -80$. No appreciable correlation could be found between the x- and y-components.[2] As a whole, the representation is valid for the whole range in x and y and shows no discontinuities. No definite trace of any moving groups could be found from the velocity diagram.

A similar study was made of the velocities projected on the xz-

[1] A slight correlation of the type mentioned above would be introduced by the fact that the errors in the absolute magnitudes are the same for the three velocity components.

[2] The absence of a linear correlation ($\overline{xy} = \bar{x}\bar{y}$) simply means that the velocities are projected on the principal axes.

and yz-planes. It was found in all cases that no dependence existed between the velocity components, apart from a slight correlation due to the fact that the errors in the absolute magnitudes are the same for all directions. In general, the high velocities are as well represented as the smaller velocities, and no discontinuity could be found.

8. The values found for the centers of the three symmetrical functions may be compared with the values of α', β', and γ' derived in *Contribution* No. 293, page 20, from the radial velocities of the stars in general.

We have

$$\alpha' = -10.1 \pm 0.5 \text{ km/sec.}; \qquad c_x = -8.8 \pm 0.7 \text{ km/sec.}$$
$$\beta' = -10.0 \pm 1.0 \text{ km/sec.}; \qquad c_y = -9.0 \pm 1.0 \text{ km/sec.}$$
$$\gamma' = -\ 4.1 \pm 0.3 \text{ km/sec.}; \qquad c_z = -1.6 \pm 0.6 \text{ km/sec.}$$

Except for the value of c_z, which as may be seen from Figure 3 is difficult to determine, the agreement is satisfactory, but there is a systematic difference which will be discussed in connection with the solar motion. The asymmetry constant here determined may be regarded as a vector starting at this point (the "limiting center") and directed toward the negative y-axis ($\alpha = 130°$, $\delta = -57°$) and has a length of 0.0206 sec./km. We may also use the inverse value, 48.5 km/sec., as a measure of the asymmetry vector. This is to be compared with the value 52.1 km/sec. corresponding to $p = 0.0192$ found in *Contribution* No. 293. This is probably the best method of expressing the asymmetry vector.

9. We will now consider the solar motion as derived from the giant M stars. On account of the asymmetry in the velocity distributions, the group motion loses some of its usual meaning; we shall retain the expression, however, supposing it to represent the arithmetical mean of the velocity components, while the solar motion will be the opposite vector. Since there is no appreciable asymmetry along the x- and z-axes, we may identify the mean for these axes with the center of the distribution. Comparing the means for the apparent and true distributions, we see that for the x- and z-components the effect of the accidental errors of observation upon the

mean values is very small. The computation of the true mean for the y-components is complicated by the fact that the computed effect of the errors near the maximum of the distribution-curve is not correct on account of the discontinuity in the derivatives. But, without appreciable error, we may regard the mean of the apparent distribution for the y-components also as representing the mean of the true distribution.[1] The three means are thus

$$\bar{x}=-9.2\pm1.3\ ;\quad \bar{y}=-17.4\pm0.9\ ;\quad \bar{z}=-2.3\pm0.8\ \text{km/sec.}$$

The opposite vector, referred to equatorial co-ordinates, gives the following value for the sun's motion referred to the giant M stars:

$$A_0=281^\circ.7\ ;\quad D_0=+35^\circ.5\ ;\quad V_0=19.8\pm1.0\ \text{km/sec.}$$

From the radial velocities of 307 M stars brighter than apparent magnitude 6.5, the following elements were derived for the solar motion:

$$A_0=279^\circ.1\ ;\quad D_0=+35^\circ.2\ ;\quad V_0=23.4\pm1.6\ \text{km/sec.}$$

Although the two apices agree well, there is an appreciable difference in the sun's velocity. This difference is intimately connected with the fact mentioned in *Contribution* No. 327[2] that the parallactic motions give a smaller value for the mean parallaxes, and thus a brighter mean absolute magnitude than the peculiar motions, provided we use the higher value for the sun's velocity.

The ratio of the two values for the sun's velocity would indicate that the absolute magnitudes need a systematic correction of $\Delta M=-0.57$. Such a correction is difficult to reconcile with the result of the comparison between angular and linear peculiar motions. Systematic errors in the proper motions may be partly responsible for this difference. A small excess of stars of high radial velocity in the southern hemisphere would account for part, or even the whole, of the effect. It must be noted, however, that a correction $\Delta M=-0.57$ would not appreciably modify the shape of the three distribution-curves here given. The only effect would

[1] Assuming the frequency function to be valid up to the singular point, we find for the true mean $\bar{y}=-18.20$ km/sec.

[2] *Astrophysical Journal*, **65**, 108, 1927.

be to change the scale of the velocities, counted from the sun as origin, by the factor $23.4/19.8 = 1.18$. All quantities expressed in km/sec. would be multiplied by this factor; quantities expressed by pure numbers (S, E, γ_4) would remain unaffected.

10. There has been some discussion about the nature of the asymmetry, and a few remarks may remove some misconceptions. Oort[1] has studied practically the same stars as those discussed in *Contributions* Nos. 245, 275, and 293,[2] and arrived at the conclusion that the asymmetry exists only for stars having a peculiar velocity greater than a certain fixed limit, which he places at 63 km/sec. Stars with peculiar velocities smaller than this limit are supposed to show no asymmetry. This asymmetry extends over a whole hemisphere, but no provision is made for effecting the transition to the other hemisphere, or for expressing the change in the limit as the axis of preferential motion is approached. No analytical expression for the velocity distribution of the high-velocity stars has been given, apart from the statement that they must lie outside a certain sphere in velocity space. This, however, would be of less importance, if any fixed discontinuity could actually be found. But the distribution curve in Figure 2 gives no trace of any such discontinuity at Oort's limit, or at any other limit. In the two-dimensional distribution, Table II, there is no indication of any discontinuity separating the high-velocity stars from those of smaller velocity. From both the one- and the two-dimensional velocity distributions we may conclude that the asymmetry is a phenomenon which affects the velocity distribution of the giant M stars as a whole.

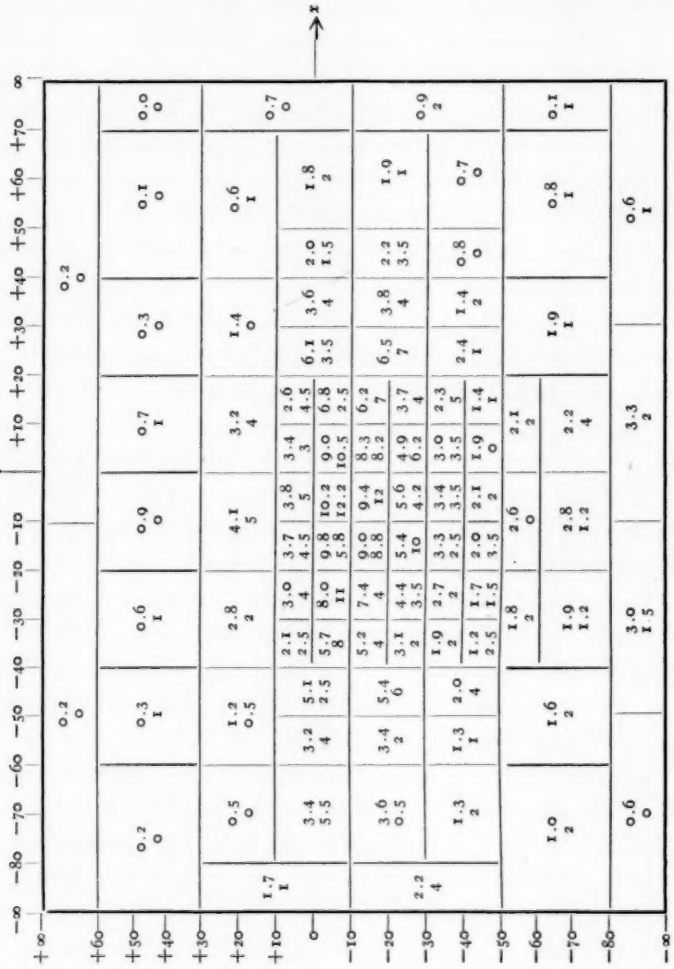
In *The Observatory*, October, 1926, Oort writes, "Most astronomers, Strömberg also, will agree that there is a pronounced *change* in the velocity distribution near 62 km/sec., in the figure on page 4 of *Groningen Publications* No. 40." To this the writer would say that the stars there studied are very heterogeneous as regards motion, and that for such a mixture we should expect a change even on the basis of the present theory. If we mix together, say, Cepheids of long and short period, we should certainly find a pronounced dis-

[1] *Bulletin of the Astronomical Institutes of the Netherlands*, No. 23, 1922, and *Groningen Publications*, No. 40, 1926.

[2] *Astrophysical Journal*, **56**, 265, 1922; **59**, 228, 1924; **61**, 379, 1925.

continuity and, if the two classes of objects are in nearly the same proportion, also an additional maximum in the velocity distribution. The same applies, although to a lesser extent, if we mix together

TABLE II



stars selected on the basis of apparent brightness and large proper motion. Since large radial velocities are correlated with large linear tangential motions (at least in a heterogeneous mixture), we obtain, in studying the radial motions, the equivalent of two or more different velocity ellipsoids, as in *Contribution* No. 293; and the

transition between these velocity ellipsoids, or error-curves in the one-dimensional case, may easily show at least a break in the continuous run. The F stars of large reduced proper motion and of large radial velocity, for instance, probably will produce a change in the distribution somewhere around Oort's limit. For this reason my more homogeneous data in *The Observatory*, September, 1926, are more satisfactory as a test of the existence of the fixed discontinuity than Oort's larger numbers, which were also previously studied by me. It is also for this reason that the giant M stars are well suited to test this question.

Some doubts have been expressed by Oort upon the general validity of the quadratic relationship between group motion and velocity dispersion, but although there is some scattering of the points on my parabola in *Contribution* No. 293 the general tendency is unmistakable. In fact, the M stars fit the parabola rather poorly; but we now find, when the three-dimensional motions are studied, that the asymmetry is the same as for the stars in general. The division into several velocity groups made in *Contribution* No. 293 does not mean, as has been conjectured, that there is no asymmetry within the individual groups; in fact, the asymmetry may be rather pronounced within the component groups, the dissection serving merely as an artifice to determine the asymmetry vector. The separation into velocity groups could have been used in the present case for the distribution of the y-components, in which case the distribution would have been represented by at least two error-curves with quite different dispersions, one having its center at about -10 and the other at about -70 km/sec. But such a representation would have been rather artificial, and, in the present case, in view of the non-existence of any correlation between x and y, without any justification. Using Oort's method, we would have expressed the same distribution by an error-curve and, in addition, an extremely asymmetrical distribution starting rather abruptly at a fixed point. It must be borne in mind, however, that Oort's representation refers primarily to the three-dimensional distribution; but as seen in Table II there is no justification for introducing any discontinuity or separation into two velocity groups of more or less complex nature. The problem would have been somewhat different

were there dynamical reasons for expecting a strictly Maxwellian distribution of stellar velocities; then we should have to invent special causes to explain the excess of high velocities.

The asymmetry vector seems to be the same for all known cosmical objects, although there is some doubt about the spiral nebulae, principally on account of the large K-term. The phenomenon is complicated by the existence of moving groups, like the Taurus and the Ursa Major groups, which probably must be treated as individual stars. The B stars show a decided discrepancy, but we have reason to believe that the bright B stars form a separate system (the "secondary galaxy," or the "local system" of Shapley), and may be comparable to a moving cluster. Several of the groups in *Contribution* No. 293 selected on the basis of small reduced proper motions (i.e., high intrinsic brightness) seem to include a large percentage of stars belonging to this local B-star system.

From a comparison of mean parallaxes computed from the luminosity and density functions with those derived from parallactic motions, Seares[1] has concluded that the sun's velocity is larger for intrinsically faint stars than for stars of higher luminosity. This result has been objected to by van Rhijn,[2] Oort and N. W. Doorn,[3] and P. van de Kamp.[4] There may be some connection between the phenomenon found by Seares and the general asymmetry in stellar motions. My own results indicate not a direct dependence of solar motion upon absolute magnitude but rather upon velocity dispersion, and this can well be reconciled with Oort's and Doorn's data. Van de Kamp finds that the radial velocities of stars of apparent magnitudes 9–10 give not only the same solar motion, but also about the same velocity dispersion as the brighter stars of the same spectra. Although seldom explicitly stated, everybody seems to have found a larger dispersion and larger solar motion for the giant K stars than for A, F, and G stars.

On account of the correlation between mean peculiar motion and absolute magnitude, we should expect a correlation such as

[1] *Mt. Wilson Contr.*, No. 281; *Astrophysical Journal*, **60**, 50, 1924.

[2] *Groningen Publications*, No. 37, p. 11, 1925.

[3] *Bulletin of the Astronomical Institutes of the Netherlands*, No. 89, 1925.

[4] *Ibid.*, No. 112, 1926; *Lick Observatory Bulletin*, **12**, 88 (No. 374), 1926.

Seares finds. The apparent contradictions can, however, all be reconciled by making the assumption that the sun's velocity varies with the velocity dispersion (or, in Oort's terminology, with the percentage of high-velocity stars), and it is only when there is a marked correlation between this dispersion (relative percentage) and the absolute magnitude that we may expect any correlation between absolute magnitude and solar motion. The determination of the latter correlation requires, however, a large amount of observational data, in particular for dwarf stars, which form such a large percentage when dealing with the stars as a whole, as Seares has done.

The "limiting center" was defined as an unattainable limit for the group motion of a class of objects as its velocity dispersion approaches zero. When using the distribution function in equation (19), this name becomes somewhat inadequate; but if we use equation (12) in *Contribution* No. 293, we see that this limit is the center of one of the two symmetrical functions composing the general distribution function, the other center defining the "world-frame."

It has been objected that the "velocity restriction[1] in a universal world-frame" which can be deduced from, but may not be a necessary consequence of, the form of the general distribution law can hardly be expected to have any effect when the velocities are as small as those found in stellar motions. But we must consider that a star is not a simple object, but is composed of electrons and atomic nuclei, and that the electrons within the stars may have velocities nearly equal to that of light. If the mass of an electron were a function of its velocity relative to the world-frame, and if the classical theory of compounding velocities were used, we should expect such a restriction. Further, we are here dealing with effects accumulated during the whole lifetime of a star, and the slightest lack of symmetry in the impulses would produce an appreciable effect. Although it is simplest to think of an ether in the classical sense as the cause of the velocity restriction, this conception is probably too mechanistic, and must be replaced by something which can serve as a substratum for electro-magnetic and inertio-gravitational phe-

[1] The word "damping" has been suggested to me by several physicists, but as this term implies a physical rather than a statistical property the original term has been retained.

nomena. The general tendency is to regard the material universe not as an ensemble of cosmical objects but rather as a single entity with physical properties and irregularities (atomic nuclei and electrons).

For transverse motions the simplest way to explain the connection and coincidence between the inertial and optical frame, on the one hand, and the stellar frame, on the other, seems to be the existence of such a velocity restriction; and it was considerations of this kind which led me to try the new type of distribution law.[1] Meanwhile, many new data have been added, and detailed studies have been made of the velocity distribution. The form suggested for the velocity distribution has been found to represent the general distribution with a much higher accuracy than was ever anticipated.

The physical basis of Oort's theory is that certain stars have velocities higher than that of escape, which of course is perfectly reasonable. The reason, however, that these velocities are not distributed equally in all directions, at least in the galactic plane, is that these stars are interlopers belonging to a larger system, relative to which our local system has a high velocity. The larger system with which they are connected is supposed to be the system of globular clusters. We should then expect that these high-velocity stars would form a system statistically at rest, relative to the larger system, and that the sun's motion, relative to the system of high-velocity stars, would be around 250 or 300 km/sec. This, however, is not at all the case. According to Oort, a clustering seems to prevail outside a fixed limit in velocity space, and for higher velocities, the number of stars decreases rapidly, only a very few attaining velocities equal to that of the system of globular clusters.

Since it is definitely proved that the high-velocity stars do not have the same group motion as the globular-cluster system, they can hardly be regarded as interlopers belonging to this system. Nevertheless, there is a connection, and this connection may be expressed formally by introducing a constant asymmetry vector, and physically, or rather conceptually, by a velocity restriction in a universal world-frame; and the introduction of this vector makes it

[1] *Mt. Wilson Communication*, No. 84; *Proceedings of the National Academy of Sciences*, **9**, 312, 1923.

possible to express the velocity distribution of practically all cosmical objects in a very simple form.

Another physical explanation of the asymmetry has been put forward by B. Lindblad.[1] According to his theory, a number of different classes of objects occupy the same space, forming subsystems having their common center of mass at a distance of about 18,000 parsecs in the direction of galactic longitude 325°. The period of rotation is different for the different subsystems, the system of globular clusters, to which the galactic clouds are supposed to belong, having only a small or zero rotation; the different speeds of rotation are in the same order as the sequence of velocity ellipsoids in *Contribution* No. 293. Such a system, according to Lindblad, may remain stable for a very long time, as the encounters of the stars have very little influence on their motions. The difficulty with this theory, as the writer sees it, is that it introduces hypotheses about the motions and distribution of cosmical objects which can hardly be tested by comparison with present observational data. As far as known at present, the center of the galactic stellar system is at a distance of only 1000–2000 parsecs. Further, it is very difficult to understand how such a system of intermingled subsystems can ever have evolved from a primordial mass of gas or dust. Nevertheless, it seems to require a distribution law of velocities practically identical with the one used here.

After all, hardly anything is known of the ultimate reason for the fact that objects of different spectral types, and even of different periods of light variation, have quite different velocities. Certain important factors seem to be missing from our theories of stellar motions; and if we look carefully enough we may even find that space itself has properties—in addition to those which we ordinarily include among inertio-gravitational and electro-magnetic phenomena—which are essential for an understanding of the problem.

MOUNT WILSON OBSERVATORY
March 1927

[1] *Arkiv för Matematik, Astronomi och Fysik*, **19***A*, Nos. 21, 27, and 35; **19***B*, No. 7, 1925–1926.

MINOR CONTRIBUTIONS AND NOTES

NOTE ON PROFESSOR ROSS'S PAPER ENTITLED: "PHOTOGRAPHS OF MARS, 1926"

ABSTRACT

After alluding to confirmation by Ross of the author's observations of 1924, attention is called to a possible misunderstanding of certain references to these observations.

In the November issue of this *Journal* (64, 243) Professor Ross publishes some very interesting notes on a series of photographs of Mars made during the recent opposition. Professor Ross's observations are very similar to some made at the preceding opposition by myself, and, as those earlier results were somewhat novel, it is gratifying to see them generally confirmed by one who is at once so resourceful an observer and so competent a photographer as Professor Ross. His observations of the inequality in diameter of violet and red images, as photographed with two different telescopes and with a form of enlarging apparatus quite different from the one which I used, will go far to establish this inequality as a fact of photographic observation, whatever its explanation may be.

Perhaps I may be permitted to call attention to what appears to me to be an incompleteness in one of Professor Ross's references to my work. In concluding his discussion of the clouds shown in his photographs he states that "during the opposition of 1924, Wright, at the Lick Observatory, obtained an extensive series of photographs of Mars in the violet and ultra-violet (*Lick Observatory Bulletin*, No. 366, 1925), showing obscuring formations similar to those just noted, *which he described as the 'impermanent features' on the planet's surface.*" I have taken the liberty of emphasizing by italics the concluding phrase as it is the part of the statement with which I am especially concerned. Now it is quite true that I used the term "impermanent configurations" in *introducing* a circumstantial account of these and related phenomena, but the quoted comment seems hardly to be an adequate description of my treatment of them.

Furthermore, from Ross's use of the word "surface," without qualification, one unfamiliar with my paper might infer that I regarded them as belonging to the solid surface of the planet. As these cloudlike objects seem not to have been adequately described previously to the publication of my paper it is perhaps worth while to attempt here to avert a misapprehension of what I really said about them. By citing the *Lick Observatory Bulletin* No. 366 Professor Ross has formally provided the reader with the means for righting himself in this respect, but the paper referred to is rather long and technical, and, owing to the circumstance that in it observations and inferences based upon them are separately treated, comments on the clouds occur in rather widely separated places, and, consequently, might not readily be found. I venture, therefore, to note the following passages in *Lick Observatory Bulletin* No. 366: the subsection numbered "2" on pages 50 and 51; the paragraphs beginning at the bottom of page 58; the one beginning at the end of page 59; and paragraph 5 of page 61. A reading of these will show that I regarded the objects under discussion as atmospheric phenomena, and specifically interpreted them as "clouds," though the difference in visibility by light of different colors of these and aqueous clouds of the earth's atmosphere, which is noted as well by Ross, was remarked, and in adopting the term "clouds" attention was drawn to it. So far Professor Ross and I appear to be in agreement. I am not convinced, however, as he seems to be, of their aqueous composition, and am inclined to believe that their physical state differs from that of ordinary terrestrial clouds. Finally, I am disposed toward an explanation of their high visibility by light of short wave-lengths which has little relation to the one which he has suggested.

A fairly complete record of the clouds shown on Professor Ross's photographs is available on the Mount Hamilton plates for the current opposition, and they will be discussed in conjunction with a phenomenon of somewhat different character in a forthcoming number of the *Lick Observatory Bulletin*.

W. H. WRIGHT

MOUNT HAMILTON, CALIF.
April 13, 1927

COMMENT ON PROFESSOR WRIGHT'S NOTE

ABSTRACT

Misapprehensions as to originality in photographing planets in different wave-lengths are corrected, and a short historical summary given of the principal researches in this field.

As Professor Wright has very kindly shown me the foregoing note before publication, I take the opportunity to correct certain misapprehensions which have arisen, especially in the daily press, and in the writings of at least one popularizer of astronomy, as to the novelty, both of method and results, described by me in the paper to which Professor Wright refers. The only novelty to which I can lay claim is in certain features of the enlarging camera, which I believe to be in the nature of an improvement. On page 246 of the paper in question I say: "It is evident that we have at hand a powerful tool for the study of conditions above the surface of Mars." This might be interpreted as meaning that the methods I used were novel. I take this opportunity of disclaiming any such intention. My paper was written for astronomers, who are perfectly familiar with the methods which I have used. I was fortunate in obtaining pictures of what seemed to me an unusually brilliant and extensive cloud in the atmosphere of Mars, which I described much as I would have described a fine new canal, without claiming originality for the canal idea or for the method adopted.

It might not be out of place very briefly to review the history of the method. It is of course generally known that photographs of a planet, with yellow filter interposed, agree with the visual appearance of the surface. In *Popular Astronomy*, **33**, 440, 1925, Professor William H. Pickering reproduces some pictures which he made of Mars in 1888 and 1890, using blue-violet light, showing differences from the visual appearance, which he rightly ascribed to the atmosphere. The work was not followed up, and appears to have been forgotten. So far as I know, the first serious attempt to photograph the planets in light of a number of wave-lengths, from ultra-violet to infra-red, was by Professor R. W. Wood, at the Mount Wilson Observatory, in 1916. His experiments began in 1909. In 1912 he applied the method to the moon, at the Princeton Observatory.[1] He however made no photographs of Mars, concerning which he says:

[1] *Astrophysical Journal*, **36**, 75, 1912; *ibid.*, **43**, 310, 1916.

In view of the interesting results obtained with these planets, it is my hope that similar observations will be made on Mars, on the occasion of its next near approach to the earth. I made one photograph of the planet with ultra-violet light in October, with the 60-inch reflector, but the disk was too small to show much of interest.

Since 1907, I believe, Mars has been photographed at the Lowell Observatory, in two colors, yellow and blue-violet. In *Popular Astronomy*, **29**, 69, 1921, E. C. Slipher contributes an important paper on the Martian clouds of 1920. He says: "An outstanding feature of the observations of Mars at the 1920 opposition was the unusual, extensive, and conspicuous cloudlike forms seen on the surface."

In *Popular Astronomy*, **33**, 593, 1925, Slipher describes a cloud formation which was visible on negatives taken with yellow light, but not on those taken with blue-violet light. This appears to be an exception to the general rule, which is undoubtedly of significance.

So far as I know, the only serious work on Mars with color filters from ultra-violet to infra-red was by Wright, at the Lick Observatory, in 1924, who continued the work during the 1926 opposition. Both on account of the intensive and specialized character of Wright's work, and on account of the results of real value which have flowed from it, the credit is undoubtedly his for applying a method of attack on the planet Mars which takes its place alongside the visual, spectrographic, and radiometric methods which are being used with such success. I do not think it too much to say that more data have been accumulated on the surface, atmospheric, and temperature conditions of Mars during the last two oppositions than in the totality of preceding oppositions. While it is true that much of the data is conflicting, it is not hopelessly so. In a few years, no doubt, it will be harmonized, leading to a fascinating picture of physical conditions on our little neighbor.

The term "impermanent feature," relieved of its context, is apt to be misleading. This is due to advance in our knowledge of the planet. Fair-minded astronomers no longer question the contention of close students of the surface of the planet, that the so-called "permanent features" are not really permanent, but are subject to seasonal variations. We must accordingly distinguish carefully between surface and atmospheric changes, both of which are disclosed

by photographs taken in different wave-lengths. It is fortunate that the intensive application of the method which in effect removes the atmosphere, or leaves it on, at will, initiated by Wright in 1924, happened contemporaneously with a new and virile attack on the planet by the combined forces of visual, spectrographic, and radiometric observers.

FRANK E. ROSS

YERKES OBSERVATORY
April 19, 1927

REVIEWS

Astronomy. By JOHN CHARLES DUNCAN. New York: Harper & Bros., 1926. Pp. xiii+384. 64 plates, 174 figs. $3.75.

As a result of the recent rapid advance in knowledge of the stellar universe, several astronomers connected with undergraduate teaching faculties have felt an irresistible urge to write textbooks. Although statistics are not at hand it seems that astronomy is finding a place among the elective cultural studies, such as philosophy and sociology, in the modern college curriculum. Two types of courses for beginners in astronomy are being offered; a short course completed in one quarter or semester, and a long course of twice the length. Some teachers require a prerequisite in mathematics; others do not.

The teacher of a class in astronomy is restricted in the selection of material to be presented by the type of students in the class and the length of the course. Likewise, the writer of a textbook must select his material and determine his style of presentation by the needs of the student for whom he writes.

In the judgment of the reviewer Professor Duncan's textbook in astronomy is best suited to a class taking a long course having a prerequisite of Freshman mathematics and elementary physics. He has evidently attempted to meet the needs of those taking a short course by placing many paragraphs in finer print, thus suggesting that they may be omitted.

The value of the text is greatly enhanced by the large number of excellent reproductions of astronomical photographs.

In his own teaching Professor Duncan has supplemented his classroom and textbook work by numerous exercises and observational problems, but he has included none of this material in his text.

The history of astronomy is no small part of the history of civilization. Professor Duncan has skilfully associated all of the great discoveries in astronomy of the past and present with the names of the men who made them. This is in harmony with the best practice in science-teaching today.

The facts of astronomy can be marshaled to give an impressive demonstration that the heavenly bodies have a history which extends

backward in time so far that we cannot distinguish it from eternity. In other words, they have passed through an evolution. But in most cases we are quite in the dark as to the beginning and early stages of that evolution. It seems to the reviewer a mistake to devote any time or space to the presentation of theories which in our present knowledge must be regarded as highly speculative. Professor Duncan has been fairly conservative in his treatment of the theories of cosmic evolution, giving only about ten pages to that phase of astronomy. The reviewer objects particularly to the prominence given to George Darwin's theory of tidal evolution of the earth and moon, without calling attention to any alternative hypothesis.

The author is to be commended for including a short chapter on the telescope, and for not omitting charts of the constellations. Unfortunately, however, these charts are photographic reproductions of much larger ones with the result that the Greek letters and some of the names are illegible.

In future editions the author may wish to correct the following misprints:

P. 200, revise statement about Mercury.
284, *change* ξ Orionis *to* ζ Orionis
287, *change reference from* Plate 13.4 *to* Plate 17.3
384, *in Index, change* Struve H. (1854–) *to* Struve, W. (1793–1864)
384, *in Index, under "Sun," change* 261 *to* 161
Plate 17.6, *change* ϕ *to* ζ *in the title*

WILLIAM O. BEAL

Investigations on the Theory of the Brownian Movement. By ALBERT EINSTEIN. Edited with notes by R. FÜRTH; translated by A. D. COWPER. New York: E. P. Dutton & Co., 1926. Pp. 119. $1.75.

This volume is a reprint of the main original papers published by Einstein on Brownian movements between 1905 and 1911. In the Appendix and notes, Dr. Fürth has added some expansions and explanations of the original text as well as historical remarks and references to the literature since 1911. The volume will be welcomed by students of theoretical physics and chemistry who have some familiarity with the elementary treatment of the subject, and desire to read the mathematical theory as originally developed by Einstein.

A. J. DEMPSTER